The Kerr/CFT Correspondence

Holography for Real World Black Holes

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+ work in progress with Song and Strominger

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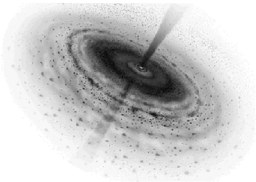


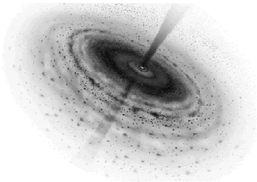
Holography and Black Holes

Motivation 1: BH Information Paradox

- 1960s: Black Hole “Mechanics”
- 1970s: Hawking radiation / Thermodynamics
  - BH evolution is non-unitary in effective field theory.
- 1990s: AdS/CFT
  - Dual CFT description is clearly unitary

~~Problem solved??~~

Conclude: The low energy description of string theory / quantum gravity is not what it seems, i.e. local effective QFT + general relativity.



Motivation 2: Observations of Black Holes

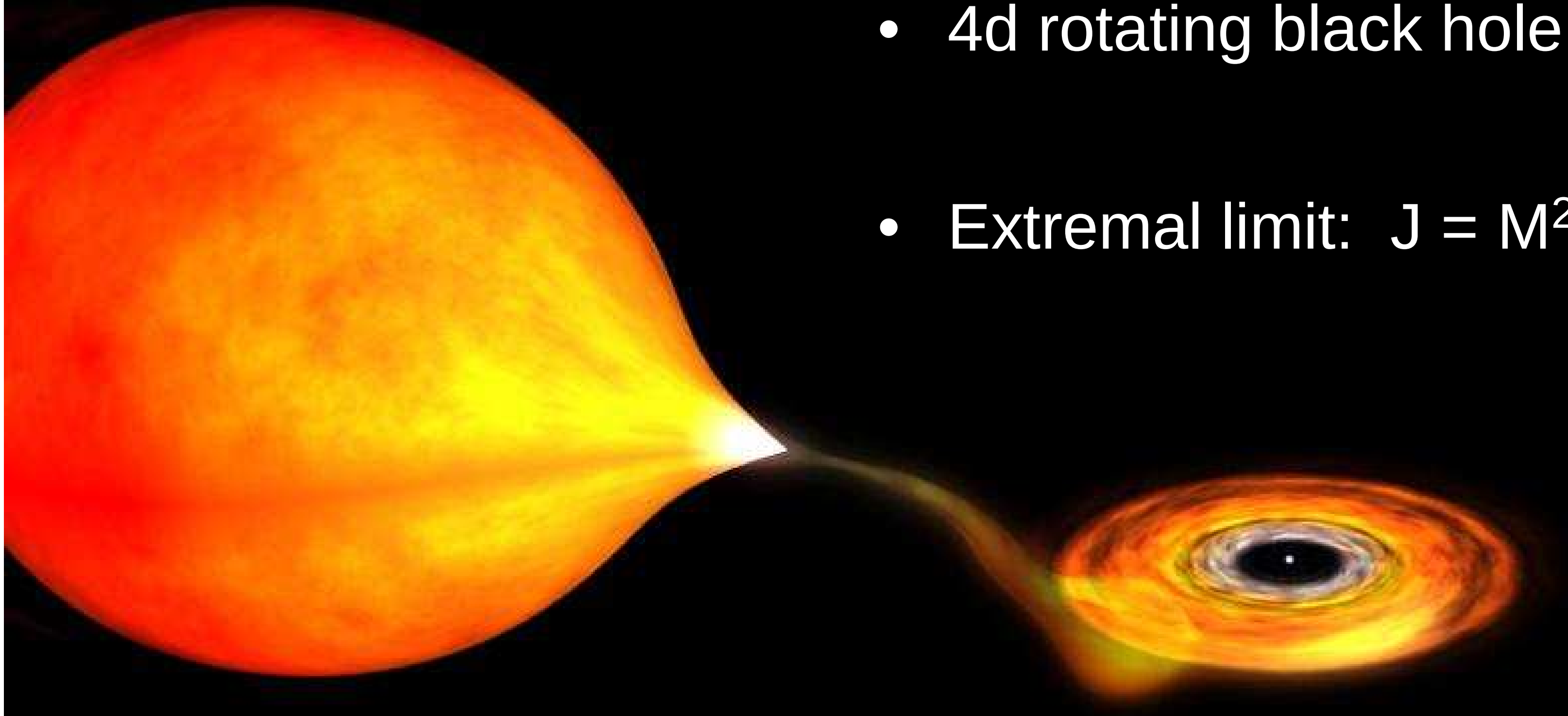
- Holography has led to a better understanding of black holes in string theory (SUSY, extra dimensions, etc.)

- But what can we learn about real-world black holes observed in the sky?

Kerr Black Holes

- 4d rotating black hole

- Extremal limit: $J = M^2$

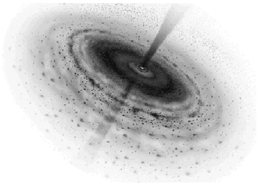


- GRS 1915+105: $J \sim .99M^2$

McClintock et al. 2006

- Bekenstein-Hawking Entropy

$$S_{\rm ext} = \frac{\rm Area}{4} = 2\pi J$$



Punch line

- Main Result

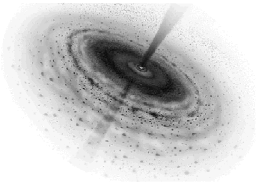
  Near the horizon of an extremal Kerr black hole, any consistent theory of quantum gravity is dual to a 2D conformal field theory.

  Central charge: $c = 12\ J$

- Derivation: states transform under a Virasoro algebra (ie in representations of the 2d conformal group)

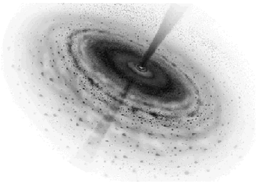
- Applies to astrophysical black holes (and more)

- Things we don't need
  - Charge
  - Anti de Sitter space (AdS)
  - Extra dimensions
  - Supersymmetry
  - String theory



Outline

- Overview ✓
- Asymptotic Symmetries
- Entropy
- Generalizations and applications
    - Charge
    - Anti de Sitter space (AdS)
    - Extra dimensions
    - Supersymmetry
    - String theory



AdS_3/CFT_2

- Explains every entropy calculation in string theory, eg entropy of 5d black holes Strominger, Vafa ‘95

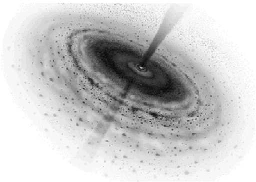
- But, complexities of string theory are not needed
Strominger ‘97

- Brown & Henneaux (’86) showed *quantum gravity on AdS_3 is dual to a CFT with central charge*

$$c = \frac{3\ell}{2G}$$

ℓ = AdS radius
G = Newton constant

- Method: Asymptotic Symmetry Group (ASG)



Near horizon extreme Kerr (NHEK)

Near horizon limit:

$$ds^2 = 2J\Omega^2\left(\underbrace{-r^2dt^2+\frac{dr^2}{r^2}}_{AdS_2}+d\theta^2+\Lambda^2(d\phi+rdt)^2\right)$$

Bardeen, Horowitz ‘99

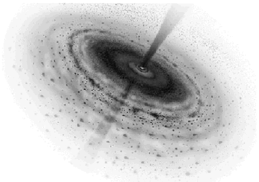
$\Omega^2,\ \Lambda^2 =$ functions of θ

$\phi \sim \phi + 2\pi$

Isometries:

$U(1)_L$ rotating ϕ

$SL(2,R)_R$ acting on the AdS_2

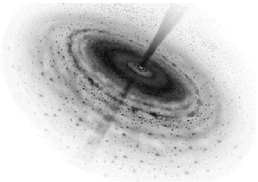


Asymptotic Symmetries I

- Asymptotic Symmetry Group [example: $U(1)$ gauge theory]

$$\text{ASG} = \frac{\text{Allowed symmetries}}{\text{Trivial symmetries}}$$

- "Allowed" = obeying the boundary conditions
- "Trivial" = corresponding charge vanishes



Asymptotic Symmetries II

- Find allowed diffeos:

$$\begin{aligned} \zeta_t &= \partial_t \\ \zeta &= \epsilon(\phi)\partial_\phi - r\epsilon(\phi)\partial_r \end{aligned}$$

- Generators ζ_n with $\epsilon_n = e^{in\phi}$ satisfy a Virasoro algebra,

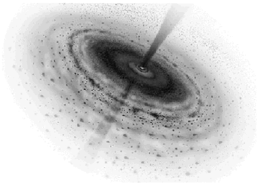
$$i\{\zeta_m, \zeta_n\}_{L.B.} = (m-n)\zeta_{m+n}$$

- Associated charges $Q_n(g_{\mu\nu})$ are boundary integrals

Determined by action

$$Q(\zeta, g) = \int_{\partial\Sigma} k[\zeta, g]$$

- Supplemental boundary condition $M^2 = J$ (extremality)



Central charge

- ★ Compute Dirac brackets

$$\{Q_m, Q_n\}_{D.B.} = \delta_n Q_m$$

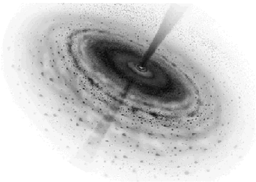
- ★ Result is the Virasoro algebra,

$$i\{Q_m, Q_n\}_{D.B.} = (m-n)Q_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n}$$

$\Rightarrow$ quantum gravity on NHEK is holographically dual to a 2d CFT with

$$\boxed{\mathrm{c} = 12\ \mathrm{J}}$$

$$\text{GRS } 1915{+}105 \rightarrow c \sim 10^{79}$$

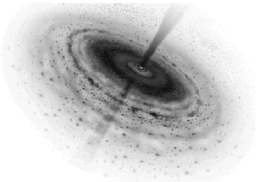


Outline

- Overview ✓
- Asymptotic Symmetries ✓
- Entropy
  - Cardy formula

$$S = \frac{\pi^2}{3} cT$$

- Generalizations and applications



Temperature

- At extremality, first law of thermodynamics becomes

$$0 = T_H dS = dM - \Omega_H dJ$$

- So define conjugate potential for extremal variations

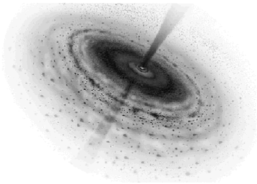
$$dS = \frac{dJ}{T_L}$$

- For Kerr,

$$S = 2\pi J \to T_L = \frac{1}{2\pi}$$

- Quantum state of a field on extreme Kerr has density matrix

$$\rho = e^{-\hat{J}/T_L}$$



Entropy

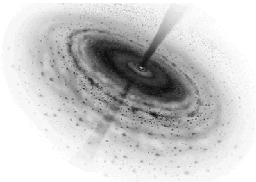
Plug central charge and temperature

$$\begin{aligned} c_L &= 12J \\ T_L &= \frac{1}{2\pi} \end{aligned}$$

into the Cardy formula

$$S_{CFT} = \frac{\pi^2}{3} c_L T_L$$

$$S_{CFT} = \frac{2\pi J}{\hbar} = \frac{\text{Area}}{4} = S_{macro}$$



Near-extremal entropy

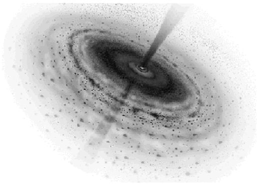
- ★ If we assume

$$c_R = c_L = 12J$$

then the Cardy formula gives the correct near extremal entropy,

$$S_{CFT} = 2\pi J + 2\pi\sqrt{\frac{c_R}{6}E_R} + \cdots$$

- ★ Summary: We have only found the chiral left half of the CFT, but there is evidence for right-movers which account for the entropy away from extremality



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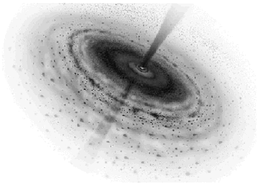
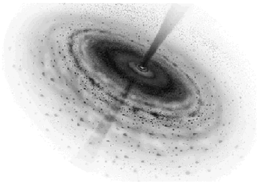
What can we compute?



Other black holes

- 4d Kerr **Guica, TH, Song, Strominger**
- Higher dimensions **Lu, Mei, Pope**
- Asymptotic AdS **various papers**
- Charge **TH, Murata, Nishioka, Strominger**
- String theory (D0-D6, D1-D5, NS5) and Supergravity
  Azeyanagi, Ogawa, Terashima
  Nakayama
  Chow, Cvetic, Lu, Pope
  Lu, Mei, Pope, Vazquez-Poritz
  Chen, Wang



Greybody Factors

- Extreme Kerr has $T_H = 0$, but it decays via superradiance into modes

$$\Phi \sim e^{im\phi - i\omega t} S_\ell(\theta) R(r)$$

with

$$0 < \omega < m\Omega_H$$

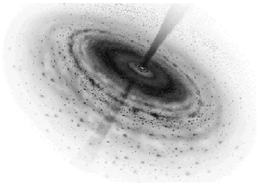
For small ω,

$$\text{Decay rate} = \Gamma_\ell(\omega) \sim (\omega - m\Omega_H)^{2\ell+1}$$

- This is a two-point function in the CFT

$$\Gamma \sim \int e^{-i\omega_R x^+ - i\omega_L x^-} < \mathcal{O}\mathcal{O} >$$

similar to:
Maldacena, Strominger ‘97



Greybody Factors

- What about large frequency?

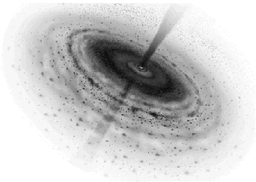
$$\Gamma = \frac{\sinh^2 2\pi\delta}{\cosh^2 \pi(m-\delta)+\cosh^2 \pi(m+\delta)+2\cos 2\pi\sigma \cosh \pi(m+\delta) \cosh \pi(m-\delta)}$$

$$\delta \equiv \text{function of } m,\ \ell,\ M$$

- Gravity: Teukolsky and Press 1974

- CFT: work in progress!

  with W. Song and A. Strominger

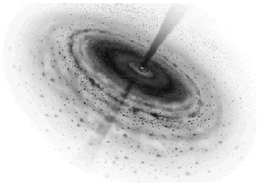


Conclusion

- **Summary: Gravity on extreme Kerr is a CFT.**
  - Nothing exotic is necessary (but exotic black holes work too)
  - Applies to astrophysical black holes, eg GRS 1915+105

- **Open questions**
  - Beyond extremality
  - What can we calculate with the CFT?
    - greybody factors?
    - astrophysics (accretion, X-ray emission, etc.)?

- **Wide open questions**
  - What is the CFT?
  - What/where are the microstates?



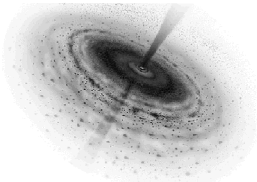
What about SL(2,R)?

AdS$_3$ (Brown-Henneaux)

$$\begin{tikzcd} \text{Exact:} & SL(2,R)_L \arrow[d] & \times & SL(2,R)_R \arrow[d] \\ \text{Asymptotic:} & \text{Virasoro} & \times & \text{Virasoro} \end{tikzcd}$$

Kerr

$$\begin{tikzcd} \text{Exact:} & U(1)_L \arrow[d] & \times & SL(2,R)_R \arrow[d] \\ \text{Asymptotic:} & \text{Virasoro} & \times & ??? \end{tikzcd}$$



What about SL(2,R)?

Near-extremal entropy

- The zero mode of SL(2,R)$_R$ is

$$\zeta_0 = \partial_t$$

- Writing this in terms of the original Kerr coordinates suggests

$$Q_0 \sim M^2 - J \equiv E_R$$

- If we assume

$$c_R = c_L = 12J$$

  then the Cardy formula gives the correct near extremal entropy,

$$S_{CFT} = 2\pi J + 2\pi \sqrt{\frac{c_R}{6} E_R} + \cdots$$

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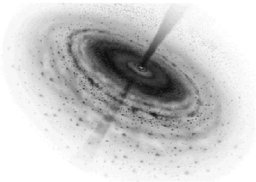

Assumptions

- For Kerr/CFT (“quantum gravity on NHEK is a CFT”), only assumption is:
  - A consistent UV completion of quantum gravity on NHEK exists
- For entropy, using the Cardy formula assumes:
  - Modular invariance
  - Sufficient but not necessary condition:

$$T \gg c \quad (\text{ie}, \ \frac{1}{2\pi} \gg 10^{79})$$

Uh-oh.

Same thing happens in string theory, but is explained by highly twisted sectors. Does something similar happen here?

Maybe – the mass gap is very small ~ 1/M^3. This suggests an effective description with small c, large T. More on this later.

“U-duality”

- ★ 5D 3-charge black hole

$$S = 2\pi\sqrt{n_1 n_2 n_3}$$

- ★ String theory U-duality changes c, T with $S \propto cT$ fixed

- ★ 5d Kerr (or 4d Kerr-Newman) has near horizon isometries

Lu, Mei, Pope

TH, Murata, Nishioka, Strominger

$$SL(2,R)_R \times U(1)_\phi \times U(1)_\psi$$

- ★ Two consistent choices of boundary conditions:
  - ▶ First choice: $U(1)_\phi \rightarrow$ Virasoro with central charge

$$c_\phi \sim J_\phi$$

  - ▶ Second choice: $U(1)_\psi \rightarrow$ Virasoro with central charge

$$c_\psi \sim J_\psi$$

- ★ *Either* choice gives the correct entropy!