PINNED BRANES AND NEW LORENTZ NON-INVARIANT THEORIES

- HEP-TH/0002175
- WITH SHOIBAL CHAKRAVARTY
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PLAN OF THE TALK:

- INTRODUCTION
- TENSION OF M5 BRANE
- SUPERGRAVITY BACKGROUND
- LORENTZ INVARIANCE
- SUPERGRAVITY VERIFICATION
- MASSIVE PARTICLES
- A DECOUPLED THEORY
- LARGE C LIMIT

INTRODUCTION :

CONSIDER A TAUB-NUT SPACE IN M-THEORY.

ELEVEN DIMENSIONAL COORDINATES ARE :

$$0,\ 1,\ 2,\ \ldots\ldots,\ 7,\ \underbrace{8,\ 9,\ 10}_{\mathbb{R}^3 \text{ DIRECTIONS OF TN}}$$

1 → M-THEORY 11^{th} DIRECTION

7 → TAUB-NUT CIRCLE

PRECISE STRUCTURE OF A TN SPACE :

TN METRIC IS GIVEN BY

$$ds^2 = \left(1 + \frac{R}{2|\vec{x}|}\right)^{-1}\left(dx^7 - A_i\, dx^i\right)^2 + \left(1 + \frac{R}{2|\vec{x}|}\right)\left(d\vec{x}\right.$$

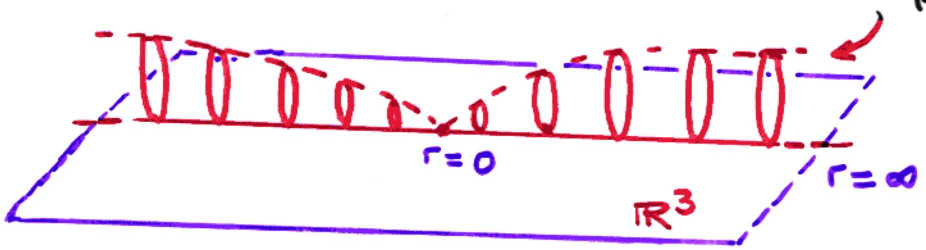
$$x^i = x^{8,9,10}$$

$$|\vec{x}| = \sqrt{(x^8)^2 + (x^9)^2 + (x^{10})^2}$$

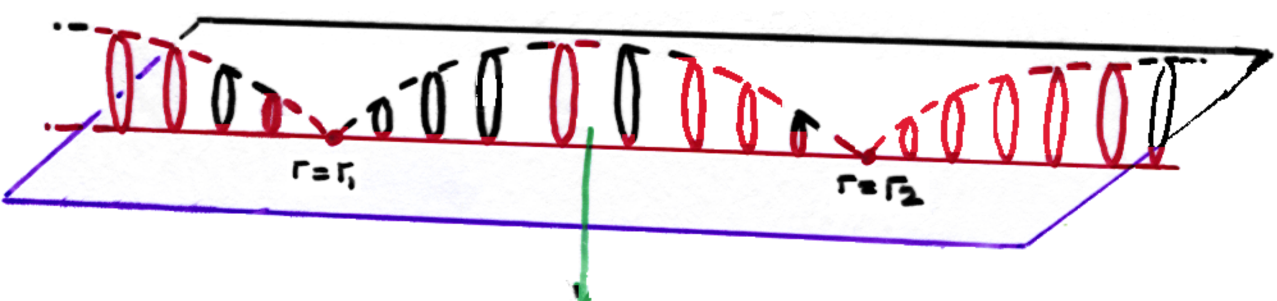
R = RADIUS OF TN AT $|\vec{x}| \to \infty$

- SINGLE CENTRED TN
- MULTI CENTRED TN

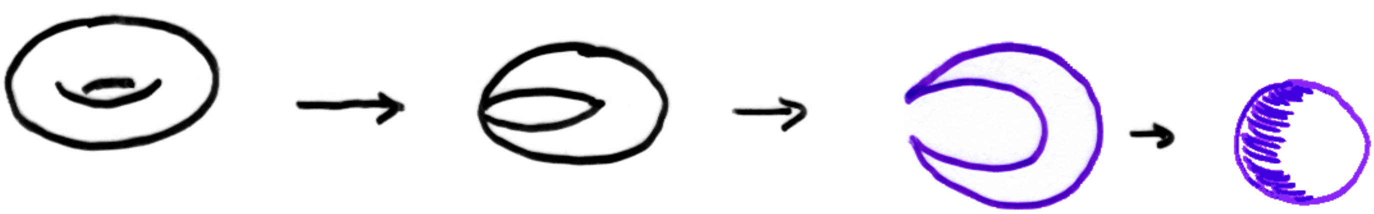
SINGLE CENTRED TN :



TWO CENTRED TN



S^2 = TORUS SHRUNK AT ONE POINT



- A TWO CENTRED TN HAS AN S^2 WHERAS A ONE CENTRED TN DOESNT HAVE
- ONE CENTRED TN HAS A HARMONIC FORM WHICH IS UNIQUE.

NEAR ORIGIN, i.e AT $r=0$, A TN SPACE BEHAVES AS:

$$ds^2 = \frac{r}{R}\left(dx^7 - A_i dx^i\right)^2 + \frac{R}{r}\left(dr^2 + r^2 d\Omega^2\right)$$

$\Rightarrow$ U(1) FIBRATION OVER BASE S^2

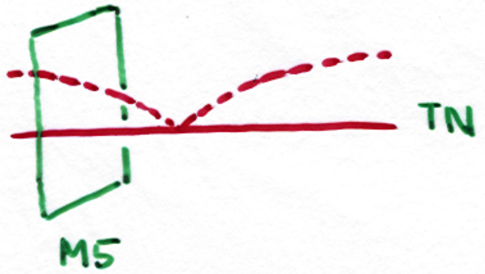
BECAUSE IF WE CHOOSE NEAR

$$|\vec{x}| \equiv r = 0 \; ; \quad r = u^2$$

WE GET

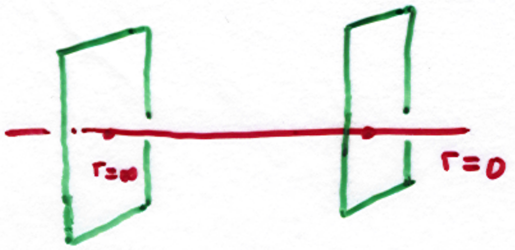
$$\frac{u^2}{R}\left(dx^7 - A_i dx^i\right)^2 + R\left(du^2 + u^2 d\Omega^2\right)$$

Q: HOW DOES A M5 BRANE BEHAVE IN A TN SPACE?



TN : x^7, x^8, x^9, x^{10}

M5 : $x^0, x^1, \ldots, x^5$

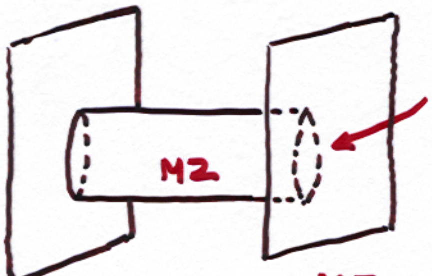


TENSION REMAINS SAME

→ PREDICTED BY BPS FORMULA

A BIT ABOUT M5 BRANE:

5ϕ : FIVE DIFFERENT DIRECTION A M5 BRANE CAN MOVE $(x^6, \ldots, x^{10})$



TO SUPPORT STRING ON THE BOUNDARY A M5 BRANE SHOULD HAVE A TWO FORM. B_{MN}.

BUT A TWO FORM IN SIX DIM$^{\underline{N}}$ CAN BE SELF-DUAL / ANTI SELF-DUAL

$(\bar{B}_{MN}, 5\phi)$: MULTIPLET OF $(2,0)$

$\downarrow$ ALSO $\qquad\qquad$ $\uparrow$ FERMIONS CHIRAL

$(\bar{B}_{MN}, \phi) + (4\phi)$

$\downarrow$ MASSLESS HYPER

NOW WE SWITCH ON A C-FIELD.

THE C-FIELD IS C_{167}

ONE LEG ALONG TN CIRCLE

ONE LEG ALONG M5

- THIS BACKGROUND BREAKS LORENTZ INVARIANCE BOTH ON BRANE AS WELL AS ON BG

- WE CHOOSE $C_{167}(r=\infty) = C =$ A CONST.

THIS DOESNOT IMPLY $G = dC$ CAN BE ZERO EVERYWHERE

- $G \neq 0$ (ALSO $C \neq$ CONSTANT) AT ANY POINT INSIDE

- METRIC, THEREFORE WILL CHANGE DRASTICALLY

Q1: CAN WE PREDICT (CALCULATE) THE METRIC ?

Q2: WHAT HAPPENS TO THE M5 BRANE TENSION ?

TENSION OF M5 BRANE

WE WILL USE THE MASSES OF BPS PARTICLES IN M-THEORY ON T^7

$V = R_1 R_2 \ldots R_7$ = VOL OF T^7

BPS PARTICLES (56 U(1) CHARGES):

R_i^{-1} , $M_P^9 V R_i$, $M_P^3 R_i R_j$, $M_P^6 V R_i^{-1} R_j^{-1}$

R_i^{-1}: KK PARTICLES (7)

$M_P^9 V R_i$: KK MONOPOLES (7)

$M_P^3 R_i R_j$: M2 BRANES (21)

$M_P^6 V R_i^{-1} R_j^{-1}$: M5 BRANES (21)

SWITCH ON A C-FIELD (CONSTANT) AT INFINITY

CENTRAL CHARGE MATRIX:

$$Z = M_P^9 V R_7 \Gamma^{78} + i N M_P^6 V R_6^{-1} R_7^{-1} \Gamma^{67} + i C_{167} M_P^6 V R_7 \Gamma^{16}$$

$M_P^9 V R_7$: TN SPACE $\equiv$ KK MONOPOLE

Γ^{78}: GAMMA MATRICES FOR SPINORS IN 8_s REPRESENTATION OF SO(8)

N: NO. OF M5 BRANES

C_{167}: BACKGROUND C-FIELD ∞.

MAXIMUM EIGENVALUE OF THE MATRIX

$$|Z_1| = M_P^6 V \sqrt{(M_P^3 R_7 + N R_6^{-1} R_7^{-1})^2 + C_{167}^2 R_7^2}$$

WE COULD ALSO CALCULATE SEPARATE MASSES OF THE TN AND THE M5 ALONE

SUM OF THE MASSES IS

$$|Z_2| = M_P^6 V \Big[\underbrace{\sqrt{M_P^6 R_7^2 + C_{167}^2 R_7^2}}_{\text{TN WITH } c} + N \underbrace{R_6^{-1} R_7^{-1}}_{\text{M5 BRANE}} \Big]$$

BOUND STATE ENERGY IS

$$M = |Z_2| - |Z_1| = N M_P^6 V R_6^{-1} R_7^{-1} \left(1 - \frac{1}{\sqrt{1+c^2}}\right)$$

$$c = M_P^{-3} C_{167} (r = \infty)$$

- TENSION OF THE M5 BRANE EFFECTIVELY DECREASES BY $\sqrt{1+c^2}$ WHEN IT IS BOUND TO THE TN

SUPERGRAVITY BACKGROUND

- WHAT WE NEED:

  N M5 BRANES $\oplus$ A TN SPACE (ONE CENTRE OR MULTICENTER) $\oplus$ A BG C-FIELD

- SYSTEM SHOULD SOLVE SUGRA EQUN OF MOTION

- ALSO CIRCLE DIRECTION OF TN IS NOT THE M-THEORY ~~R~~ CIRCLE

  $\downarrow$ THEREFORE IN IIA WE EXPECT

- A TN SPACE
- A D4 BRANE
- A $B^{(NSNS)}$ FIELD WITH LEGS ORTHOGONAL TO THE D4 BRANE

NOTE: SINCE $x^1 \equiv$ M-THEORY DIRECTION, TYPE IIA WILL HAVE DIR.

$$(x^0, x^2, x^3, \ldots, x^8, x^9, x^{10})$$

SUPERGRAVITY BACKGROUND

- WHAT WE NEED:

  N M5 BRANES $\oplus$ A TN SPACE (ONE CENTRE OR MULTICENTER) $\oplus$ A BG C-FIELD

- SYSTEM SHOULD SOLVE SUGRA EQUN OF MOTION

- ALSO: CIRCLE DIRECTION OF TN IS NOT THE M-THEORY ~~R~~ CIRCLE

$\downarrow$ THEREFORE IN IIA WE EXPECT

- A TN SPACE (x^7, x^8, x^9, x^{10})
- A D4 BRANE $(x^0, x^2, x^3, x^4, x^5)$
- A $B^{(NSNS)}$ FIELD WITH LEGS ORTHOGONAL TO THE D4 BRANE (B^{NSNS}_{67})

NOTE: SINCE $x^1 \equiv$ M-THEORY DIRECTION, TYPE IIA WILL HAVE DIR.

$(x^0, x^2, x^3, \ldots, x^8, x^9, x^{10})$

LET US TAKE A CONFIGURATION IN IIB AS:

D5	x^0	x^2	x^3	x^4	x^5	—	x^7	—	—	—
NS5	x^0	x^2	x^3	x^4	x^5	x^6	—	—	—	—

METRIC CAN BE WRITTEN DOWN EXPLICITLY

DIRECTIONS x^6, x^7 ARE ON A TORUS

METRIC IS:

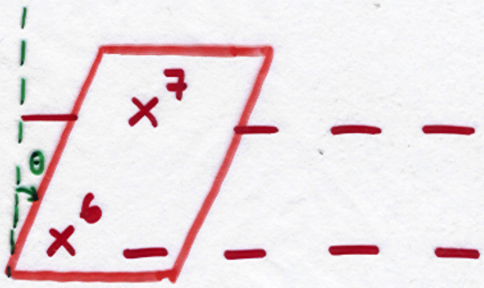
$$ds^2 = H_3^{-1/2}\, ds^2_{02345} + H_5 H_3^{1/2}\, ds^2_{89,10}$$
$$+ H_3^{1/2}\, ds_6^2 + H_5 H_3^{-1/2}\, ds_7^2$$

H_3: HARMONIC FN FOR D5 BRANES

H_5: HARMONIC FN. FOR NS5 BRANES

LET US TAKE A CONFIGURATION IN IIB AS:

D5	x^0	x^2	x^3	x^4	x^5
NS5	x^0	x^2	x^3	x^4	x^5



METRIC CAN BE WRITTEN DOWN EXPLICITLY

DIRECTIONS x^6, x^7 ARE ON A SLANTED TORUS

METRIC IS:

$$ds^2 = H_3^{-1/2}\, ds^2_{02345} + H_5 H_3^{1/2}\, ds^2_{89,10}$$
$$+ H_3^{1/2}(dy^6 \sec\theta)^2 + (H_3^{1/2}\sin^2\theta + H_5 H_3^{-1/2}\cos^2\theta)(dy^7)^2$$
$$+ 2 H_3^{1/2} \tan\theta\, dy^6 dy^7$$

MIXED COMPONENT g_{67} HAS APPEARED

A T-DUALITY ALONG x_7

NS5	$\longleftrightarrow$	TN SPACE	LIFT TO M-THEORY $\longrightarrow$	$\longleftrightarrow$	TN
D5	$\longleftrightarrow$	D4 BRANE		$\longleftrightarrow$	M5 BRANE
g_{67}	$\longleftrightarrow$	B_{67}		$\longleftrightarrow$	C_{167}

EXACTLY THE BG THAT WE REQUIRE.

THE C-FIELD BG IS

$$C_{167} = M_P^3 \frac{H_3^{1/2} \tan\theta}{\left(H_3^{1/2} \sin^2\theta + \frac{H_5}{H_3^{1/2}} \cos^2\theta\right)} dx^1 \wedge dy^6 \wedge (dy^7 + A_i dx^i)$$

$$C_{167} (r \to \infty) = \tan\theta = \text{CONSTANT}$$

CALCULATE M5 BRANE TENSION

WE HAVE TO COMPUTE $\sqrt{\det(\text{METRIC})}$ ALONG THE M5 DIRECTION

$$\sqrt{\det} = \sqrt{G_{00} G_{11} \cdots G_{55}}$$

$$= (H_5^{-1} \sin^2\theta + \cos^2\theta)^{1/2}$$

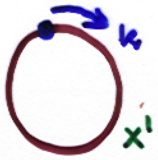
- AT $r \to \infty$ $\sqrt{\det} = 1$ (ALL $H_i = 1$)
- NEAR $r \to 0$

$$\sqrt{\det} = \frac{1}{\sqrt{1+c^2}}$$

EXPECTED FROM BPS CALCULATIONS

LORENTZ INVARIANCE (?)

PARTICLE PROPAGATING ALONG x^1



$$|Z_1| = \underbrace{M_P^9 V R_7 \Gamma^{78}}_{\text{TN}} + \underbrace{i N M_P^6 V R_6^{-1} R_7^{-1} \Gamma^{67}}_{\text{M5}} + \underbrace{i C_{167} M_P^6 V R_7 \Gamma^{16}}_{\text{GFIELD}} + \underbrace{i \frac{k}{R_1} \Gamma^{18}}_{\text{PARTICLE WITH MOM}^{\text{M}}\ k}$$

MASS OF PART. $= |Z_1| - |Z_2| = \dfrac{k}{\sqrt{1+c^2}} R_1^{-1}$

$$= k \tilde{R}_1^{-1}$$

WHERE $\tilde{R}_1 \equiv \sqrt{1+c^2}\, R_1$

FOR PARTICLE MOVING ALONG x^2, MASS IS $k R_2^{-1}$

$\therefore$ BY RESCALING R_1 TO $\tilde{R}_1$, ENERGY OF A MASSLESS PARTICLES WITH MOMENTUM P IS GIVEN BY LORENTZ INV. VALUE $|P|$

DOES THAT MEANS THAT WE CANNOT DETECT THE LOR. NONINV. BY MASSLESS PART?

SUPERGRAVITY VERIFICATION

LET $h^{-1} \equiv H_3^{1/2} \sin^2\theta + H_5 H_3^{-1/2} \cos^2\theta$

SUGRA BG IN M-THEORY IS:

$$ds^2 = (h H_5)^{2/3} dx_1^2 + (h H_5)^{-1/3} ds^2_{02345}$$
$$+ (h H_5)^{2/3} (dy^6)^2 + h^{2/3} H_5^{-1/3} (dx^7 + B_{7i} dx^i)^2$$
$$+ h^{-1/3} H_5^{2/3} ds^2_{8,9,10}$$

$\downarrow$ NEAR $r \to 0$
(AFTER A SCALING BY $(1+c^2)^{1/3}$

$$ds^2 = (1+c^2) dx_1^2 + dx^2_{02345} + (1+c^2) dx_6^2$$
$$+ (1+c^2) \frac{r}{R} (dx_7 + A_i dx_i)^2 + \frac{R}{r}(dr^2 + r^2 d\Omega^2)$$

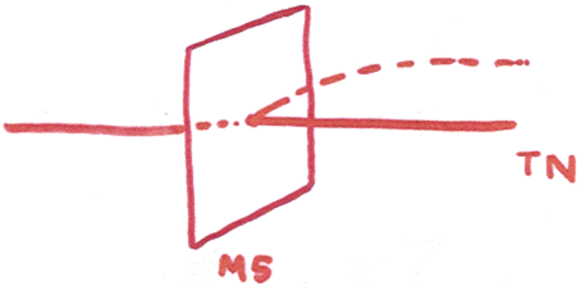
IF WE SCALE

$$\tilde{x}_1 \equiv \sqrt{1+c^2}\; x_1$$

THEN SO(5,1) LORENTZ ALONG M5 IS RESTORED

- AT LOWEST ORDER BREAKING OF SO(5,1) LORENTZ INVARIANCE WILL BE REGISTERED BY THE FERMIONS

MASSIVE PARTICLES :



M5 BRANE STUCK AT $r=0$

$\Rightarrow$ ANY SMALL FLUCTUATION WILL BE GIVEN BY A MASSIVE FIELD

MASS OF BPS STATE CHARGED UNDER THE U(1) ISOMETRY OF TN

$$Z_1 = \left(M_p^9 V R_7 + \underbrace{ik\, R_7^{-1}}_{\text{MOMENTUM}}\right)\Gamma^{78} + i N M_p^6 V R_6^{-1} R_7^{-1} \Gamma^{67} + i M_p^6 C_{167} V R_7 \Gamma^{16}$$

Z_2 = PUTTING $k=0$

ENERGY OF EXCITATION $= |Z_1| - |Z_2|$

$$= k \frac{c}{\sqrt{1+c^2}} R_7^{-1}$$

FROM SUGRA:

EXPAND $\sqrt{\det}$ IN SMALL r.

$$\sqrt{\det} = \frac{1}{\sqrt{1+c^2}} + \frac{c^2}{2(1+c^2)^{1/2}} M_P r + O(r^2)$$

$\downarrow$ $u^2 = r$, GOOD COORD NEAR ORIGIN

$$\int dx_0\, dx_1 \cdots dx_5 \, \frac{1}{2} \cdot \frac{c^2}{(1+c^2)^{1/2}} M_P u^2$$

$\downarrow$ $\tilde{x}_1 = (1+c^2)^{1/2} x_1$

$$\frac{1}{2} m^2 u^2 = \frac{1}{2} \frac{c^2}{(1+c^2)} M_P u^2$$

MASS OF THE SCALARS: $\dfrac{c}{(1+c^2)^{1/2}}$

SUMMARY OF THE RESULTS

- IF WE RESCALE

$$R_1 \longrightarrow \sqrt{1+c^2}\, R_1$$

THEN NEAR $r=0$ SO(5,1) INV IS RESTORE[D]

$\Rightarrow$ MASSLESS PARTICLES ALL HAVE LORENTZ INV MOMENTA $|p|$

- TENSION OF A M5 BRANE IS LOWER BY A FACTOR OF $\sqrt{1+c^2}$ AT THE CENTER OF THE TN (RELATIVE TO INFINITY)

- SPECTRUM HAS FOUR MASSIVE SCALAR PARTICLES WITH MASS $\dfrac{c}{(1+c^2)^{1/2}}$

- M5 BRANES ARE SEPARATED THERE

APPEAR TO BE STRINGS IN THE SPECTRUM

A DECOUPLED THEORY :

VARIOUS ENERGY SCALES IN THE THEORY

- M_P IS THE PLANCK SCALE
- R_7^{-1} SCALE OF KK EXCITATIONS FAR AWAY FROM THE CENTER
- $M_P c^{1/3}$ IS THE ENERGY SCALE OF BINDING ENERGY/VOLUME
- $c R_7^{-1}$ IS THE ENERGY SCALE OF EXCITATIONS OF THE M5-BRANE

$M_P \to \infty$ DECOUPLE GRAVITY

$M_P c^{1/3} \to \infty$ M5 IS PINNED AT ORIGIN

$R_7^{-1} \to \infty$ DECOUPLE KK OSCILLATIONS

$c R_7^{-1} \to$ FINITE $\Rightarrow c \to 0$ (CASE WE CONSIDERED)

NEW (1,0) SIX DIM THEORY

LOW ENERGY DESCRIPTION :
(2,0) THEORY WITH MASSIVE HYPERMULTIPLET.

A SIX DIMENSIONAL THEORY WITH (2,0) SUSY DOESN'T SUPPORT A MASSIVE HYPERMULTIPLET (BECAUSE OF CHIRALITY)

$$\underset{\substack{\uparrow \\ (2,0)}}{(\bar{B}, 5\phi)} \neq \underset{\substack{\uparrow \\ (1,0)}}{(\bar{B}, \phi)} + \underset{\substack{\uparrow \\ \text{HYPER (MASSIVE)}}}{(4\phi)}$$

→ LORENTZ NON INVARIANCE COMES TO OUR RESCUE!

$$SO(5,1) \xrightarrow[\text{BY } C_{167}]{\text{BROKEN}} SO(4,1)$$

$SO(4,1)$: FERMIONS CAN BE GIVEN MASS

- MASS SHOULD BE PROPORTIONAL TO C
  $\therefore$ $C \to 0$ WE RECOVER (2,0)
- MASS OF FERMIONS = MASSIVE BOSONS (BY SUSY)

SUGRA MECHANISM FOR MASS GENERATION

M-THEORY TERM

$$\int d^{11}x \sqrt{G} \; \underbrace{\bar{\psi}^M}_{\text{GRAVITINO}} \; \Gamma^{PQ} \; \psi^N \; \underbrace{F_{MNPQ}}_{\text{FOUR FORM dc}}$$

$$F_{MNPQ} = F_{167r} = c(1+c^2)$$

$$\bar{\psi}^M \Gamma^{PQ} \psi^N F_{MNPQ} \rightarrow \bar{\psi}^7 \Gamma^{16} \psi^r F_{167r}$$

$$\bar{\psi}^7(x^0, x^1, \ldots, x^{10}) = \underbrace{\bar{\lambda}}_{\text{COLLECTIVE COORDINATE}}(x^0, x^1, \ldots, x^5) \; \underbrace{\psi_0^7}_{\text{ZERO MODE}}(x^6, \ldots, x^{10})$$

$$\int d^{11}x \sqrt{G} \; \bar{\psi}^M \Gamma^{PQ} \psi^N F_{MNPQ}$$

$$\rightarrow \underbrace{\bar{\lambda} \Gamma^{16} \lambda}_{\text{MASS TERM FOR FERMIONS}} \int d^6x \underbrace{\sqrt{G} \; \bar{\psi}_0^7 \; \psi_0^r \; F_{167r}}_{\text{MASS}}$$

MASS $\propto F_{167r} = c(1+c^2)$?

RESOLUTION :

- ZERO MODE $\psi_0^7(x^6, \dots, x^{10}) = \dfrac{f_1(x^6, \dots, x^{10})}{\sqrt{1+c^2}}$

NORMALISABLE FN.
FOR $C_{167} = 0$

- $\psi_0^r = f_2(x^6, \dots, x^{10})$

- $dx^1 \equiv \dfrac{d\tilde{x}^1}{\sqrt{1+c^2}}$

AND ALSO $dx_6 \equiv \dfrac{d\tilde{x}^6}{\sqrt{1+c^2}}$

$$\int d^6x \sqrt{G}\, \bar{\psi}_0^7 \psi^r F_{167r} = \frac{c}{\sqrt{1+c^2}} \int d^6x \sqrt{G}\, f_1 f_2$$

$\Rightarrow$ MASS OF THE FERMIONS $\propto \dfrac{c}{\sqrt{1+c^2}}$

: SAME AS BOSONS

LARGE C LIMIT:

- ASSUME NOW TN x^7 CIRCLE
  = M-THEORY CIRCLE

- REMOVE M5 BRANE FROM THE PICTURE

TN $\longrightarrow$ D6 BRANE

$C_{167} \longrightarrow B_{16}$

$\Downarrow$

A NON COMMUTATIVE 6+1 D THEORY ON THE SIX BRANE.

Q: WHAT ARE THE BPS STATES OF THIS THEORY?

KK PARTICLES:

$$\sqrt{\frac{k_1^2}{\tilde{R}_1^2} + \frac{k_2^2}{R_2^2} + \cdots + \frac{k_5^2}{R_5^2} + \frac{k_6^2}{\tilde{R}_6^2}}$$

M2 BRANES:

$$\frac{M_P^3}{c} R_I R_J \,, \quad I,J = 2,3,4,5$$

$$\frac{M_P^3}{c} \tilde{R}_I R_J \,, \quad I = 1,6 \,, \quad J = 2,\ldots,5$$

M5 BRANES

$$\left(\frac{M_P^3}{C}\right)^2 \tilde{R}_1 R_2 R_3 R_4 R_5$$

ELECTRIC FLUXES

- ALONG 1 : $\frac{M_P^3}{C} \tilde{R}_1 R_7$ ← CONFINED ?
- ALONG 2 : $\left(\frac{M_P^3}{C}\right)^{-1} \frac{R_2}{\tilde{R}_1 R_3 R_4 R_5 \tilde{R}_6}$

MAGNETIC FLUXES ARE ALSO PROPORTIONAL TO $\left(\frac{M_P^3}{C}\right)$

⇒ IF WE KEEP THE FOLLOWING LIMITS

$$M_P \to \infty$$

$$C \to \infty$$

$$\frac{M_P^3}{C} \to \text{FIXED}$$

WE GET A 6+1 D NCYM THEORY WITH FINITE MASS BPS OBJECTS.

g^2_{YM} OF OUR THEORY

$$= \frac{M_P^3}{C} = \text{FIXED}$$

- THIS IS PRECISELY THE SEIBERG–WITTEN LIMIT.

- $M_P \to \infty$ THEORY TEND TO BE DECOUPLED BUT HAS CONTINUUM OF STATES LIKE A 10+1 D THEORY

CONCLUSIONS :

- FOR SMALL C, TENSION OF A M5 BRANE CAN BE REDUCED TO $\frac{T_\infty}{\sqrt{1+c^2}}$
- SCALING $x_1 \to \sqrt{1+c^2}\, x_1$ RESTORES $SO(5,1)$ LORENTZ INVARIANCE

  BUT TO LOWEST ORDER FERMIONS BREAK IT
- FOR A SPECIFIC SCALING OF M_P AND R_7 (AND SMALL C) WE GET A DECOUPLED THEORY WHOSE LOW ENERGY LIMIT IS A (2,0) THEORY WITH A MASSIVE HYPER.
- FOR LARGE C WE GET (IN $d=10$) A NON COMMUTATIVE YM THEORY WHOSE BPS EXCITATIONS ARE FINITE.

  IN PARTICULAR WE FIND CONFINED ELECTRIC FLUX.
- OUR WORK MIGHT HELP US UNDERSTAND HOW TO FIX A BRANE IN AN AMBIENT SPACE.