PINNED BRANES AND NEW LORENTZ NON-INVARIANT THEORIES

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PLAN OF THE TALK:

- INTRODUCTION
- TENSION OF M5 BRANE
- SUPERGRAVITY BACKGROUND
- LORENTZ INvariance
- SUPERGRAVITY VERIFICATION
- MASSIVE PARTICLES
- A DECOUPLED THEORy
- LARGE C LIMIT
INTRODUCTION:

CONSIDER A TAUB-NUT SPACE IN M-THEORY.

ELEVEN DIMENSIONAL COORDINATES ARE:

0, 1, 2, ..., 7, 8, 9, 10

M-THEORY

11th DIRECTION

TAUB-NUT CIRCLE

IR^3 DIRECTIONS OF TN

PRECISE STRUCTURE OF A TN SPACE:

TN METRIC IS GIVEN BY

\[ ds^2 = \left( 1 + \frac{R}{2|x^1|} \right)^{-1} (dx^7 - A_i dx^i)^2 + \left( 1 + \frac{R}{2|x^1|} \right) (dx^1)^2 \]

\[ x^i = x^8, 9, 10 \]

\[ |x^1| = \sqrt{(x^8)^2 + (x^9)^2 + (x^{10})^2} \]

\[ R = \text{RADIUS OF TN AT } |x^1| \to \infty \]

· SINGLE CENTRED TN
· MULTICENTRED TN
SINGLE CENTRED TN:

TWO CENTRED TN

$S^2 = \text{Torus shrunk at one point}$

- A two centred TN has an $S^2$ whereas a one centred TN doesn't have
- One centred TN has a harmonic form which is unique.
Near origin, i.e. at $r = 0$, a TN space behaves as:

$$ds^2 = \frac{r}{R} (dx^9 - A_i dx^i)^2 + \frac{R}{r} (dr^2 + r^2 d\Omega^2)$$

$\Rightarrow$ $U(1)$ fibration over base $S^2$

Because if we choose near $(x^9) \equiv r = 0$; $r = u^2$

we get

$$\frac{u^2}{R} (dx^9 - A_i dx^i)^2 + R (du^2 + u^2 d\Omega^2)$$

Q: How does a M5 brane behave in a TN space?

TN: $x^9, x^8, x^9, x^{10}$

M5: $x^0, x^1, \ldots, x^5$

Tension remains same

$\Rightarrow$ predicted by BPS formula
A BIT ABOUT M5 BRANE:

(2,0) theory \((B^\mu_\nu, 5\phi)\)

along \((x^0, x^1, \ldots, x^5)\)

\(5\phi\): FIVE DIFFERENT DIRECTION A M5 BRANE CAN MOVE \((x^6, \ldots, x^{10})\)

END POINT IS A STRING

TO SUPPORT STRING ON THE BOUNDARY
A M5 BRANE SHOULD HAVE A TWO FORM \(B^{MN}\).

BUT A TWO FORM IN SIX DIM. CAN BE
SELF-DUAL / ANTI SELF-DUAL

\((B^M_N, 5\phi)\): MULTIPLE OF \((2,0)\)

\(\downarrow\) ALSO

\((\tilde{B}^M_N, \phi) + (4\phi)\)

\(\downarrow\) FERMIONS CHIRAL

MASSLESS HYPER
Now we switch on a C-field.

The C-field is $C_{167}$

- One leg along the circle
- One leg along M5

- This background breaks Lorentz invariance both on brane as well as on BG

- We choose $C_{167} (\tau = \infty) = C = \text{a const.}$

This does not imply $G = dC$ can be zero everywhere.

- $G \neq 0$ (also $C \neq \text{constant}$) at any point inside

- Metric, therefore will change drastically

Q1: Can we predict (calculate) the metric?

Q2: What happens to the M5 brane tension?
TENSION OF M5 BRANE

WE WILL USE THE MASSES OF BPS PARTICLES IN M-THEORY ON T^7

V = R_1 R_2 ... R_7 = VOL OF T^7

BPS PARTICLES (56 U(1) CHARGES):

\[ R_i^{-1}, \quad M_0 V R_i, \quad M_3 R_i R_j, \quad M_6 V R_i^{-1} R_j^{-1} \]

\[ \downarrow \quad \text{KK MONOPOLES} \quad \downarrow \]

\[ \text{KK PARTICLES} \quad (21) \quad \text{M2 BRANES} \quad (21) \quad \text{M5 BRANES} \quad (21) \]

SWITCH ON A C-FIELD (CONSTANT) AT INFINITY

CENTRAL CHARGE MATRIX:

\[ Z = M_0^9 V R_7 \Gamma^{98} + i N M_6 V R_7^{-1} R_7^{-1} \Gamma^{67} \]

\[ \downarrow \quad \text{TN SPACE} \quad \downarrow \quad \text{NO. OF} \quad \text{M5 BRANES} \quad \text{BACKGROUND} \quad \text{C-FIELD} \quad \infty \]

\[ = \text{KK MONOPOLE} \quad \downarrow \quad \text{GAMMA MATRICES FOR SPINORS} \quad \text{IN 8_8 REPRESENTATION OF SO(8)} \]
MAXIMUM EIGENVALUE OF THE MATRIX

\[ |Z_2| = M_p^6 V \sqrt{(M_p^6 R_7 + N R_6^{-1} R_7^{-1})^2 + C_{167}^2 R_7^2} \]

WE COULD ALSO CALCULATE SEPARATE MASSES OF THE TN AND THE MS ALONE

SUM OF THE MASSES IS

\[ |Z_2| = M_p^6 V \left[ \sqrt{M_p^6 R_7^2 + C_{167}^2 R_7^2} + N R_6^{-1} R_7^{-1} \right] \]

TN WITH C \hspace{2cm} \downarrow \hspace{2cm} \text{M5 BRANE}

BOUND STATE ENERGY IS

\[ M = |Z_2| - |Z_1| = N M_p^6 V R_6^{-1} R_7^{-1} \left( 1 - \frac{1}{\sqrt{1+c^2}} \right) \]

\[ C = M_p^{-3} C_{167} (r=\infty) \]

- TENSION OF THE M5 BRANE EFFECTIVELY DECREASES BY $\sqrt{1+c^2}$ WHEN IT IS BOUND TO THE TN
SUPERGRAVITY BACKGROUND

• WHAT WE NEED:
  \[ N \text{ M5 branes} \oplus \text{a TN space (one centre or multicenter)} \oplus \text{a BG C-field} \]

• SYSTEM SHOULD SOLVE SUGRA EQUATION OF MOTION

• ALSO CIRCLE DIRECTION OF TN IS NOT THE M-THEORY CIRCLE

\[ \downarrow \text{ THEREFORE IN IIA WE EXPECT} \]

• A TN SPACE
• A D4 BRANE
• A \( B^{\text{non-Abelian}} \) FIELD WITH LEGS ORTHOGONAL TO THE D4 BRANE

NOTE: SINCE \( x^1 \equiv \text{m-theory direction} \), TYPE IIA WILL HAVE DIR.
\[ (x^0, x^2, x^3, \ldots, x^8, x^9, x^{10}) \]
SUPERGRavity BACKGROUND

- WHAT WE NEED:
  \[ N \text{ M5 branes} \oplus \text{a TN space (one centre or multicenter)} \oplus \text{a BG G-field} \]

- SYSTEM SHOULD SOLVE SUGRA EQUATION OF MOTION

- ALSO: CIRCLE DIRECTION OF TN IS NOT THE M-THEORY \& CIRCLE
  \[ \therefore \text{therefore in IIa we expect} \]

- A TN SPACE \((x^3, x^8, x^9, x^{10})\)
- A D4 BRANE \((x^0, x^3, x^3, x^{4,5})\)
- A \(B^{(nsns)}\) FIELD WITH LEGS ORTHOGONAL TO THE D4 BRANE \((B^{nsns}_{67})\)

NOTE: SINCE \(x^I = \text{M-theory direction, type IIa will have dir.} \)
\((x^0, x^3, x^3, ..., x^8, x^9, x^{10})\)
Let us take a configuration in II B as:

\[ \begin{array}{cccccc}
D5 & x^0 & x^2 & x^3 & x^4 & x^5 \\
NS5 & x^0 & x^2 & x^3 & x^4 & x^5
\end{array} \]

Metric can be written down explicitly. Directions \(x^6, x^7\) are on a torus.

Metric is:

\[
ds^2 = H_3^{-\frac{\nu_2}{2}} dS_{02345}^2 + H_5 H_3^{\frac{\nu_2}{2}} dS_{8910}^2 \\
+ H_3^{-\frac{\nu_2}{2}} ds_6^2 + H_5 H_3^{\frac{\nu_2}{2}} ds_7^2
\]

Harmonic fn. for D5 branes

Harmonic fn. for NS5 branes
LET US TAKE A CONFIGURATION IN IIb AS:

\[
\begin{align*}
    &\text{D5} & x^0 & x^2 & x^3 & x^4 & x^5 & x^7 \\
    &\text{NS5} & x^0 & x^2 & x^3 & x^4 & x^5 & x^6 \\
\end{align*}
\]

METRIC CAN BE WRITTEN DOWN EXPLICITLY. DIRECTIONS \( x^6, x^3 \) ARE ON A \( SLANTED \) TORUS.

METRIC IS:

\[
\begin{align*}
    ds^2 &= H_3^{r/2} \, ds^2_{02345} + H_5 \, H_3^{r/2} \, ds^2_{89,10} \\
    &+ H_3^{r/2} (dy^6 \sec \theta)^2 + (H_3^{r/2} \sin^2 \theta + H_5 \, H_3^{r/2} \cos^2 \theta)(dy^9)^2 \\
    &+ 2 \, H_3^{r/2} \tan \theta \, dy^6 \, dy^7
\end{align*}
\]

MIXED COMPONENT \( g_{67} \) HAS APPEARED. A T-DUALITY ALONG \( x^7 \).

\[
\begin{align*}
    &\text{NS5} \leftrightarrow \text{TN SPACE} \\
    &\text{D5} \leftrightarrow \text{D4 BRANE} \\
    &g_{67} \leftrightarrow B_{67} \\
    \text{LIFT TO} &\quad \text{M-THEORY} \\
    \text{TN} &\leftrightarrow M5 \text{ BRANE} \\
    \text{TN} &\leftrightarrow C167
\end{align*}
\]

EXACTLY THE BG THAT WE REQUIRE.
The C-field BG is

\[ C_{167} = M_p^3 \frac{H_3^{2/3} \tan \theta}{(H_3^{2/3} \sin^2 \theta + \frac{H_5}{H_3^{2/3}} \cos^2 \theta)} \ dx^1 \wedge dy^6 \wedge (dy^3 + A_i dx^i) \]

\[ C_{167} (r \to \infty) = \tan \theta = \text{constant} \]

**Calculate M5 Brane Tension**

We have to compute \( \sqrt{\det \text{(Metric)}} \) along the M5 direction.

\[ \sqrt{\det} = \sqrt{G_{00} G_{11} \ldots G_{55}} \]

\[ = (H_5^{-1} \sin^2 \theta + \cos^2 \theta)^{1/2} \]

- At \( r \to \infty \), \( \sqrt{\det} = 1 \) (All \( H_i = 1 \))
- Near \( r \to 0 \),

\[ \sqrt{\det} = \frac{1}{\sqrt{1+c^2}} \]

Expected from BPS Calculations
**LORENTZ INVARIANCE**

**PARTICLE PROPAGATING ALONG** \( x^1 \)

\[
\begin{align*}
|z_1| &= M^6 \hat{v} R_7 \Gamma^{38} + i N M^6 \hat{v} R_6 R_7^{-1} \Gamma^{67} \\
&\downarrow \quad TN \quad M5 \quad \downarrow \quad C FIELD + i \frac{k}{R_1} \Gamma^{18} \\
&\downarrow \quad PARTICLE WITH \quad MOM^2 k \\
\text{MASS OF PART.} &= |z_1| - |z_2| = \frac{k}{\sqrt{1 + c^2}} R_1^{-1} \\
&= k R_1^{-1}
\end{align*}
\]

WHERE \( \tilde{R}_1 = \sqrt{1 + c^2} R_1 \).

**FOR PARTICLE MOVING ALONG** \( x^2 \),

**MASS IS** \( k R_2^{-1} \).

\[ \therefore \text{BY RESCALING} \quad R_1 \quad \text{TO} \quad \tilde{R}_1, \quad \text{ENERGY OF} \]

**A MASSLESS PARTICLES WITH MOMENTUM** \( p \) **IS GIVEN BY LORENTZ INV. VALUE** \( |p| \)

**DOES THAT MEAN THAT WE CANNOT DETECT THE LOR. NONINV. BY MASSLESS PART?**
SUPERGRAVITY VERIFICATION

Let \( h^{-1} = H_3^{1/2} \sin^2 \theta + H_5 H_3^{-1/2} \cos^2 \theta \)

SUGRA BG IN M-THEORY IS:

\[
\begin{align*}
    ds^2 &= (h H_5)^{2/3} \ dx_1^2 + (h H_5)^{-1/3} \ ds_{02345}^2 \\
         &\quad + (h H_5)^{2/3} (dy^6)^2 + h^{2/3} H_5^{-1/3} (dx^7 + B + i \ dx^i)^2 \\
         &\quad + h^{-1/3} H_5^{2/3} \ ds_{8,9,10}^2 \\
\end{align*}
\]

Near \( r \to 0 \) (after a scaling by \((1 + c^2)^{1/3}\))

\[
\begin{align*}
    ds^2 &= (1 + c^2) \ dx_1^2 + dx_{02345}^2 + (1 + c^2) \ dx_6^2 \\
         &\quad + (1 + c^2) \frac{r}{R} (dx_4 + Ai \ dx_1)^2 + \frac{R}{r} (dr^2 + r^2 ds^2) \\
\end{align*}
\]

If we scale

\[
\tilde{x}_i' = \sqrt{1 + c^2} \ x_i
\]

then a Lorentz along M5 is restored.

- At lowest order breaking of \( SO(5,1) \) Lorentz invariance will be registered by the fermions.
MASSIVE PARTICLES:

M5 BRANE STUCK AT $r=0$  
⇒ ANY SMALL FLUCTUATION WILL BE GIVEN BY A MASSIVE FIELD

MASS OF BPS STATE CHARGED UNDER THE U(1) ISOMETRY OF TN

$$z_1 = (M_P^9\, V R_7 + i k R_7^{-1}) \Gamma^{28} + i N M_P^6 V R_6^{-1} R_7^{-1} \Gamma^{37} + i M_P^6 C_{167} V R_3 R_7^{-1} \Gamma^{16}$$

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\text{MOMENTUM}
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$$z_2 = \text{PUTTING } k=0$$

ENERGY OF EXCITATION $= |z_1| - |z_2|$

$$= k \frac{C}{\sqrt{1 + C^2}} R_7^{-1}$$
FROM SUGRA:

EXPAND $\sqrt{\det}$ IN SMALL $\gamma$.

$$\sqrt{\det} = \frac{1}{\sqrt{1+c^2}} + \frac{c^2}{2(1+c^2)^{3/2}} M_p \gamma + O(\gamma^2)$$

\[ \downarrow \]

$u^2 = \gamma$, GOOD COORD NEAR ORIGIN

\[ \int dx_0 dx_1 ... dx_5 \frac{1}{2} \cdot \frac{c^2}{(1+c^2)^{3/2}} M_p u^2 \]

\[ \downarrow \] \[ x_1 = (1+c^2)^{1/2} x_1 \]

$$\frac{1}{2} m^2 u^2 = \frac{1}{2} \frac{c^2}{(1+c^2)} M_p u^2$$

MASS OF THE SCALARS: $$\frac{c}{(1+c^2)^{1/2}}$$
SUMMARY OF THE RESULTS

• IF WE RESCALE

\[ R_1 \rightarrow \sqrt{1+c^2} \ R_1 \]

THEN NEAR \( r = 0 \) SO(5,1) INV IS RESTORED

⇒ MASSLESS PARTICLES ALL HAVE LORENTZ INV MOMENTA \( p_1 \)

• TENSION OF A M5 BRANE IS LOWER BY A FACTOR OF \( \sqrt{1+c^2} \) AT THE CENTER OF THE TN (RELATIVE TO INFINITY)

• SPECTRUM HAS FOUR MASSIVE SCALAR PARTICLES WITH MASS \( \frac{c}{(1+c^2)^{1/2}} \)

• M5 BRANES ARE SEPARATED THERE APPEAR TO BE STRINGS IN THE SPECTRUM
A DECOPLED THEORY:

VARIOUS ENERGY SCALES IN THE THEORY:

- $M_p$ IS THE PLANCK SCALE
- $R^{-1}_7$ SCALE OF KK EXCITATIONS FAR AWAY FROM THE CENTER
- $M_p C^{1/3}$ IS THE ENERGY SCALE OF BINDING ENERGY/VOLUME
- $C R^{-1}_7$ IS THE ENERGY SCALE OF EXCITATIONS OF THE M5-BRANE

$M_p \to \infty$ DECOUPLE GRAVITY

$M_p C^{1/3} \to \infty$ M5 IS PINNED AT ORIGIN

$R^{-1}_7 \to \infty$ DECOUPLE KK OSCILLATIONS

$C R^{-1}_7 \to$ FINITE $\Rightarrow C \to 0$ (CASE WE CONSIDERED)

NEW (1,0) SIX DIM THEORY

LOW ENERGY DESCRIPTION:

(2,0) THEORY WITH MASSIVE HYPERMULTIPLE.
A six dimensional theory with $(2,0)$ SUSY doesn't support a massive hyper multiplet (because of chirality)

$$(\tilde{B}, 5\phi) \neq (\tilde{B}, \phi) + (4\phi)$$

$$(2,0) \uparrow (1,0) \uparrow \text{HYPER (MASSIVE)}$$

→ Lorentz non invariance comes to our rescue!

\[ SO(5,1) \xrightarrow{\text{BROKEN}} \xrightarrow{\text{BY}} SO(4,1) \]

\[ SO(4,1) : \text{FERMIIONS CAN BE GIVEN MASS} \]

- Mass should be proportional to $C$:
  \[ C \rightarrow 0 \] we recover $(2,0)$

- Mass of fermions = Massive bosons (by SUSY)
SUGRA MECHANISM FOR MASS GENERATION

**M-THEORY TERM**

\[ \int d^{11}x \sqrt{G} \overline{\Psi}_M \Gamma^{PQ} \Psi_N F_{MNPR} \downarrow \text{GRAVITINO} \downarrow \text{FOUR FORM DC} \]

\[ F_{MNPR} = F_{167r} = c(1+c^2) \]

\[ \overline{\Psi}_M \Gamma^{PQ} \Psi_N F_{MNPR} \rightarrow \overline{\Psi}^7 \Gamma^{16} \Psi_5 F_{167r} \]

\[ \overline{\Psi}^7(x^0, x^1, \ldots, x^{10}) = \overline{\lambda}(x^0, x^1, \ldots, x^5) \Psi_0^7(x^6, \ldots, x^{10}) \downarrow \] \[ \text{COLLECTIVE COORDINATE} \downarrow \text{ZERO MODE} \]

\[ \int d^{11}x \sqrt{G} \overline{\Psi}_M \Gamma^{PQ} \Psi^N F_{MNPR} \rightarrow \overline{\lambda} \Gamma^{16} \lambda \int d^6x \sqrt{G} \overline{\Psi}_0^7 \Psi_5^r F_{167r} \]

**MASS TERM FOR FERMIONS**

**MASS**

\[ \text{MASS} \propto F_{167r} = c(1+c^2) \]
Resolution:

- Zero mode $\psi_0^r(x^5, \ldots, x^9) = \frac{f_1(x^5, \ldots, x^9)}{\sqrt{1+c^2}}$
  
- Normalisable function for $c_{167} = 0$

- $\psi_0^r = f_2(x^5, \ldots, x^9)$

- $dx^1 \equiv \frac{dx^1}{\sqrt{1+c^2}}$

  AND ALSO $dx^6 \equiv \frac{dx^6}{\sqrt{1+c^2}}$

\[ \int d^6x \sqrt{g} \, \overline{\psi}_0 \, \psi^r \, F_{16+5} = \frac{c}{\sqrt{1+c^2}} \int d^6x \sqrt{g} \, f_1 f_2 \]

$\Rightarrow$ Mass of the fermions $\propto \frac{c}{\sqrt{1+c^2}}$

: Same as bosons
LARGE C LIMIT:

- Assume now TN X^3 CIRCLE = M-THEORY CIRCLE
- Remove M5 brane from the picture

TN \rightarrow D6 brane
C_{167} \rightarrow B_{16}
\downarrow
A non commutative 6+1D theory on the six brane.

Q: What are the BPS states of this theory?

KK PARTICLES:
\sqrt{\frac{k_1^2}{R_1^2} + \frac{k_2^2}{R_2^2} + \ldots + \frac{k_5^2}{R_5^2} + \frac{k_6^2}{R_6^2}}

M2 BRANES:
\frac{M_P^3}{C} R_I R_J, \ I, J = 2, 3, 4, 5
\frac{M_P^3}{C} \tilde{R}_I R_J, \ I = 1, 6, J = 2, \ldots, 5
M5 BRANES

\[ \left( \frac{M_p^3}{C} \right)^2 \tilde{r}_1 r_2 \tilde{r}_3 r_4 r_5 \]

ELECTRIC FLUXES

- Along 1: \( \frac{M_p^3}{C} \tilde{r}_1 r_7 \leftarrow \text{CONFINED?} \)

- Along 2: \( \left( \frac{M_p^3}{C} \right)^{-1} \frac{r_2}{\tilde{r}_1 r_3 r_4 r_5 \tilde{r}_6} \)

MAGNETIC FLUXES ARE ALSO PROPORTIONAL TO \( \left( \frac{M_p^3}{C} \right) \)

⇒ IF WE KEEP THE FOLLOWING LIMITS

\[ M_p \to \infty \]

\[ C \to \infty \]

\[ \frac{M_p^3}{C} \to \text{FIXED} \]

WE GET A 6+1D NCYM THEORY WITH FINITE MASS BPS OBJECTS.
$g_{YM}^2 = \frac{M_P^3}{C} = \text{fixed}$

- This is precisely the Seiberg-Witten limit.

- $M_P \to \infty$ theory tend to be decoupled but has continuum of states like a 10+1 D theory
CONCLUSIONS:

- For small $c$, tension of a M5 brane can be reduced to $\frac{T_0}{\sqrt{1+c^2}}$
- Scaling $x_1 \to \sqrt{1+c^2} x_1$ restores SO(5,1) Lorentz invariance but to lowest order fermions break it.
- For a specific scaling of $M_0$ and $r_3$ (and small $c$) we get a decoupled theory whose low energy limit is a $(2,0)$ theory with a massive hyper.
- For large $c$ we get (in $d=10$) a non-commutative YM theory whose BPS excitations are finite. In particular we find confined electric flux.
- Our work might help us understand how to fix a brane in an ambient space.