FLUCTUATIONS OF THE GIANT GRAVITON

or

ASPECTS OF NON-COMMUTATIVITY IN ADS/CFT

A. Jevicki

with S. DAS, S. MATHUR

hep-th/0008088

/0009019

+ earlier work with

S. RAMGOOLAM + M. Mihailescu

- MALDACENA'S CORRESPONDENCE (ADS/CFT) PROVIDES A BRIDGE BETWEEN LARGE N YANG-MILLS TYPE TH. AND SUPERGRAVITY OR CLOSED STRING THEORY
- KEY FEATURES OF THIS CORRESPONDENCE ARE:

- $\frac{1}{N}$ PLAYS THE ROLE OF THE CLOSED STR. / GRAVITY COUPLING CONSTANT

- THE SPACE-TIME IS GIVEN BY

$$AdS \times S$$

WITH CURVATURE GIVEN BY

$$R \propto g_{YM}^2 N$$

VERY WELL KNOWN

- SYMMETRIES ARE THE SAME ON BOTH SIDE (REALIZED DIFFERENTLY)

- N (of SU(N)) ALSO SERVES AS A CUTOFF ON QUANTUM NUMBERS OF EMERGING SPACE-TIME [EXCLUSION P. OF MALDACENA-STROMINGER

- $\frac{1}{N}$ GIVES THE MEASURE OF NON-COMMUTATIVITY OF SPACE-TIME

PROPOSAL:

$$[AdS]_{\frac{1}{N}} \times [S]_{\frac{1}{N}}$$

↑ QUANTUM, NONCOMMUTING AdS AND S

A.J. + S. RAMGOOLAM
JHEP 9904 (1999)

TOPICS OF THIS TALK

(I) CFT

STUDY OF THE $S_N(\mathcal{M})$ ORBIFOLD

- 2d CFT WHICH IN THE LARGE N LIMIT IS RELATED TO GRAVITY COMPACTIFIED ON $\mathcal{M}$
- The RESULTING 6D GRAVITY HAS A $AdS_3 \times S_3$ BACKGROUND
- SINGULAR POINT IN MOD. SPACE BUT SUCESFULL IN ACC. FOR ENTROPY of BH
- CAN BE USED AS A LAB FOR SYSTEMATIC STUDY : FINITE N eff.

hep-th 9902059 RAMGOOLAM
9907144 MIHAILESCU
2006...

MATHUR'S TALK

THE GROUND STATE of n-TWISTED SECTOR:

$$L_0 |0,\sigma_n\rangle = \frac{n^2-1}{4n} |0,\sigma_n\rangle$$

$$J^3_0 |0,\sigma_n\rangle = 0$$

EXCITED STATES: CHIRAL PRIMARIES

$$(L_0 - J^3_0) |ch.\ prim\rangle = 0$$

$$(\bar{L}_0 - \bar{J}^3_0) |ch.\ prim\rangle = 0$$

RAISE THE $J^3, \bar{J}^3$ charge of the vacuum

$$|ch, n\rangle = \prod_{k=0}^{\frac{n}{2}-1} \psi^{+\dot{+}\dot{\alpha}\dot{a}}_{-(\frac{1}{2}+k)} \psi^{+-}_{-(\frac{1}{2}+k)} |0,\sigma_n\rangle$$

$$\left.\begin{aligned} J^3_0 &= \frac{n-1}{2} \\ L_0 &= \frac{n-1}{2} \end{aligned}\right\}$$

↑ BASIC EXAMPLE OF CH. PRIMARY

CHIRAL PRIMARIES: STATES WITH

$$\boxed{L_0 = J_0}$$

BPS STATES IN GRAVITY

- The S_N (PERMUTATION) ORBIFOLD is A CFT DUAL of 10D IIB SUGRA on:

$$AdS_3 \times S_3 \times K_3 \ (\text{or } T_4)$$

CONSISTS of

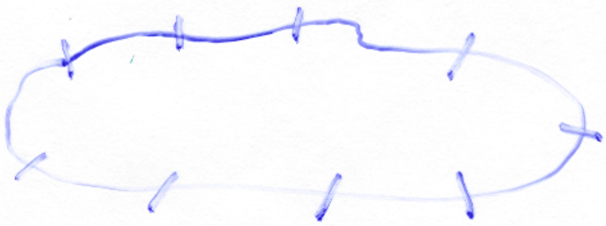
$$X_I^{a\dot{a}} \ , \ \psi_I^{a\alpha} \qquad a, \alpha = 1, 2$$

$$I = 1, \ldots N \qquad S_N \text{ SYMMETRY}$$

$$\Phi(e^{2\pi i} z) = g\, \Phi(z) \qquad \text{TWISTED B.C.}$$

n-twisted sector

$$X_{i_1}(\sigma + 2\pi) = X_{i_2}(\sigma) \ , \ X_{i_2}(\sigma + 2\pi) = X_{i_3} \quad \text{ecc.}$$



n - BITS

- Symmetry :

$$SU(1,1)_L \times S_R(1,1) \qquad L_n, \bar{L}_n \qquad \text{VIRASORO}$$

$$\times$$

$$SU(2)_L \times SU_2(R) \qquad J^a, \bar{J}^a \qquad \text{CURRENT ALG.}$$

• A SHORT DISCUSSION (WORK WITH M. MIHAILESCU

ACCORDING TO THE PROPOSAL [JR] WE ARE TO CONSIDER SUPERGRAVITY IN A q-DEFORMED BACKGROUND

$$[AdS_3]_{\frac{1}{N}} \times [S_3]_{\frac{1}{N}}$$

ONE CAN THEN REPEAT THE CALCULATION of Lee, Minwalla and Seiberg $AdS_5 \times S_5$

Mihailescu $AdS_3 \times S_3$

$$\phi = \Sigma \; \Psi(u,t,\varphi)\Big|_{AdS_3} \; \Psi(m)\Big|_{S_3}$$

The CALCULATION FACTORIZES

$$\langle \psi_1 \psi_2 \psi_3 \rangle_{AdS}\Big| \langle \psi^{l_1} \psi^{l_2} \psi^{l_3} \rangle_{S_3}$$

PROJECTED TO BOUNDARY

• FINITE N CORRELATIONS AND

(2) DEFFORMATION OF SPACE-TIME

FOR CHIRAL PRIMARY OPERATORS

WE WERE ABLE TO EVALUATE FINITE N

OPERATOR PRODUCTS (IN S_N ORBIFOLD CFT)

⇒ 3-POINT CORRELATION FUNCTIONS

FOR EXAMPLE PURE TWIST $O_l^{(0,0)}(z,\bar{z})$

$$\langle O^{\dagger}_{l_3}(\infty)\, O_{l_2}(1)\, O_{l_1}(0) \rangle \Big| \quad l_1 + l_2 = l_3$$

$$= \left[\frac{(N-l_1)!\,(N-l_2)!}{(N-l_3)!\,N!\,l_1\,l_2} \right]^{1/2}$$

• SHOWS VANISHING WHEN $l_3 = l_1 + l_2 > N$

CONFORMING WITH THE EXCLUSION P.

• EXIBITS NONCOMMUTATIVITY

$\Rightarrow$ $[S_3]_{1/N}$ gives most relevant effect

$$\psi^j(m) = D^j_{m\,m'}(\varphi, \theta, \psi)$$

STAR PRODUCT

$$\psi^{i_1}(\hat{m}) * \psi^{i_2}(m_2) = \begin{pmatrix} i_1 & i_2 & i_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} i_1 & i_2 & i_3 \\ m_1' & m_2' & m_3' \end{pmatrix}$$

$$\cdot\, F(i_1\, i_2\, i_3)_N \;\psi^{i_3}$$

FUSION COEFFICIENTS

S_3 $D^j_{m,m'=0}$

$U(1) \downarrow$

S_2 Y^j_m

$$\mathrm{Tr}\left(Y^*_{j_3 i_3}\, Y_{j_1 i_1}\, Y_{j_2 j_2} \right)$$

$$\hookrightarrow \quad \frac{(N-j_1)!\,(N-j_2)!}{(N-j_3)!} \cdot \frac{(N+j_3+1)!}{(N+j_1+1)(N+j_2+1)}$$

$j_3 = j_1 + j_2$

SAME AS IN THE ORB. — CANCEL WITH ADS

$\Rightarrow$ The structure of 3-point corr is consistent with noncomm (fuzzy) sphere

II

(SEMI) CLASSICAL ARGUMENT

McGreevy, Susskind
Toumbas

hep-th/0003075

S. DAS, A. J.
(UNPUBLISHED)
Ho + M. Li

McGreevy, Susskind and Toumbas consider the motion of the "graviton" on AdS × S space. The main effect is coming from a background n-form field strenght (for S_n)

By the Kabat-Taylor-Myers mechanism the graviton (a collection of 0-branes) grows into a spherical n-2 brane. It is the spherical n-2 brane that Susskind et al consider moving on S_m.

WE HAVE A SPHERICALLY SYMMETRIC

M-2 BRANE WITH THE RADIUS

$r(t)$ AND $\phi(t)$ (ANGLE OF ROTATION)

MOVING IN THE INDUCED METRIC

$$ds^2 = -dt^2 + \frac{R^2}{R^2 - r^2} dr^2 + (R^2 - r^2) d\phi^2 + r^2 d\Omega^2$$

θ's

AND THE NONZERO (4-FORM)

$$F_{r\phi 3 4} \propto N R r^2$$

THE INDUCED DIRAC-BORN-INFELD-C-S

LAGRANGIAN READS:

$$\mathcal{L} = -\frac{N}{R^{n-1}} r^{n-2} \sqrt{1 - \left(\frac{R^2}{R^2 - r^2} \dot{r}^2 + (R^2 - r^2) \dot{\phi}^2\right)}$$

$$+ N \left(\frac{r}{R}\right)^{n-1} \dot{\phi}$$

CLASSICAL SOLUTION :

$\dot{r} = 0 \qquad r = \text{const}$

$\dot{\phi} \neq \qquad$ NONZERO ANG. MOMENTUM

] * SPECIAL

$$J = \frac{\partial L}{\partial \dot{\phi}} = m(r)(R^2 - r^2)\gamma \dot{\phi} + N\left(\frac{r}{R}\right)^{n-1}$$

SOLVING E-L EQUATION (with $\dot{r} = 0$)

$$r \gamma \dot{\phi} \equiv x$$

$$x^2 - (n-1)x + (n-2) = 0 .$$

$$\Rightarrow J = N \left(\frac{r}{R}\right)^{n-3}$$

$$E \cdot R = J$$

} ✓

The second relation signifies that the solution describes BPS type state [CHIRAL PRIMARIES in CFT].

• The ANGULAR MOMENTUM FORMULA

$$J = N \left(\frac{r}{R}\right)^{n-3} \qquad 0 \le r \le R$$

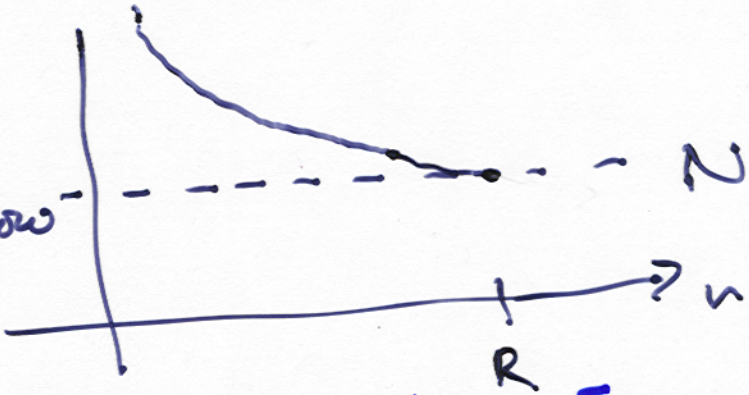
implies that for:

($n=2$) is. MOTION of $p = n-2 = 0$ BRANE ON S_2

$$J = N \frac{R}{r}$$

Bounded from below

$$J \geq N$$

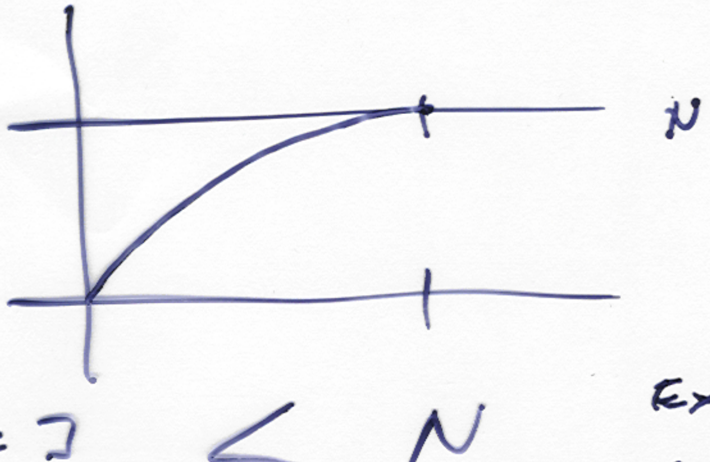


NOT VERY GOOD FOR EXCL.

($n=3$) MOTION of A DSTRING ON S_3

$$J = N \quad \text{all } r \quad ?$$

$n > 3$ MEMBRANE + higher branes — S_4 AND HIGHER



$$ER = J \leq N$$

EXCLUSION OBSERVED (FOR (M PRIN.)

- MORE GENERAL MOTIONS:

$\dot{r} \neq 0$ BOTH $r(t)$, $\varphi(t)$ TIME DEPENDENT.

Usefull to go Hamiltonian: $p_r = \frac{\partial L}{\partial \dot{r}}$

$J = P_\varphi = \frac{\partial L}{\partial \dot{\varphi}}$

$$H^2 = \underbrace{m(r)^2 + \frac{(J - \varkappa(r))^2}{R^2 - r^2}}_{V_{eff}} + \underbrace{P_r^2 \frac{R^2 - r^2}{R^2}}$$

SCALING

$p_r = \lambda \bar{P}_r$

$J = \lambda j$

$\lambda = \frac{N}{R^{n+1}}$

$$\mathcal{E}^2 = j^2 + \frac{P_r^2}{\lambda^2 g_r} + \frac{(rj - r^{n+2})^2}{1 - r^2}$$

- IMPLIES THAT FOR MOTIONS SUCH THAT $\dot{r} \neq 0$ [i.e. $\mathcal{E} = j$]

we have $\mathcal{E} > j$ AND NO BOUND ON EITHER $\mathcal{E}$ OR j

- This motions are the NONCHIRAL (STRINGY) STATES

- (DUAL) ADS GIANT GRAVITON

There is more: we have considered the brane expanding in S_n taking just time t from the AdS space-time:

$$AdS \times S$$

$$\downarrow$$

$$dt^2 + R^2 \, d\vec{X}^2$$

But one can obviosly do the opposite

$$ds^2 \Big|_{ADS} + R^2 \, d\phi^2$$

only an angle from S_n

There is clearly a spherical configuration expanding in AdS space

· See Grisaru, Myers + Tafjord / 0008015

Hashimoto, Hirano + Itzhaki / 0008016

AdS_m

$$\mathcal{L}_{m-2} = -r^{m-2}\sqrt{1+\frac{r^2}{L^2}-L^2\dot{\phi}^2} + \frac{r^{m-1}}{L}$$

$$J = p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$ – angular momentum in S_m

Two minima

$r = 0$ and $\frac{r}{L} = j^{\frac{1}{m-3}}$

· In the AdS case there is NO bound on j since r is not bounded

AdS: $\left(1+\frac{r^2}{L^2}\right)$ $\quad (1-r^2)$ of the sphere

$0 < r < \infty$

• PUZZLE (which still REMAINS)

We have three classical CONFIGURATIONS with the SAME BPS energy!

GIANT GRAVITON SPREADING IN S — $r=0$ — DUAL GIANT GRAVITON SPREADING IN AdS

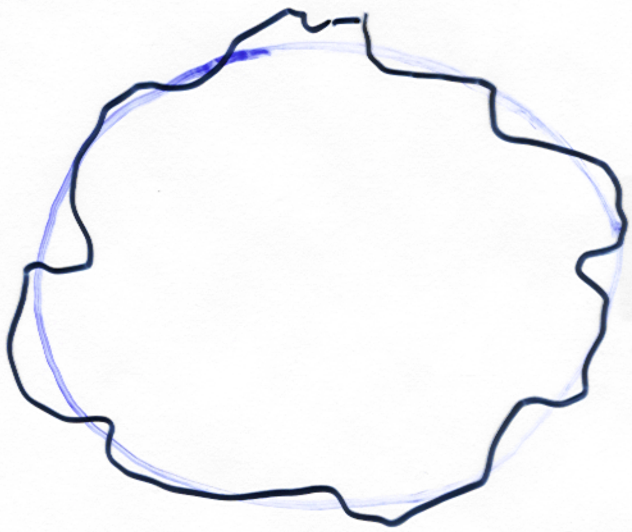
• BUT IN THE CFT THERE IS A SINGLE STATE (SINGLE Ch. PRIMARY OP) $E=J$

Resolution:

TUNELLING BETWEEN THE 3 CLASSICAL VACUA LIFTS THE ENERGY OF TWO → SINGLE STATE REMAINS

Still unclear / Myers et al / Julian Lee / 0010191

• FLUCTUATIONS of THE GIANT GRAVITON



we had a

SPHERICALY SYMMETRIC

$P_r = 0$ (STATIC)

CONFIGURATION of CONSTANT radius r_0

Question : Is this CONF. STABLE ?

SMALL FLUCTUATIONS

ALLOW fluctuATIONS iN BOTH $AdS_m \times S_m$ SPACE

PARAMETRIZATION:

$$AdS_m \qquad -(v_0^2 + v_{-1}^2) + v_1^2 + \dots v_{m-1}^2 = 1$$

$$v_0 = ch\rho \cos\frac{t}{L} \qquad t, \rho$$

$$v_{-1} = ch\rho \sin\frac{t}{L}$$

S_m

$$ds^2_{S_m} = L^2\left(1-\frac{r^2}{L^2}\right)d\phi^2 + \frac{dr^2}{1-\frac{r^2}{L^2}} + r^2\, d\Omega^2_{n-2}$$

$$r, \phi, \chi_1, \ldots \chi_{m-2}$$

CONSIDER THE $m-2$ BRANE, IN THE PHYSICAL GAUGE

$$\tau = t$$

$$\sigma_i = \chi_i \qquad i=1, \ldots n-2$$

THE REMAINING COORDINATES FLUCTUATE AND ARE GENERAL FUNCTIONS $f(t,\vec{\sigma})$

THEY ARE

$$r(t,\vec{\sigma}) = r_0 + \eta(t,\sigma)$$

$$\phi(t,\vec{\sigma}) = \omega_0\tau + \delta\phi$$

$\Big\}$ $\underline{S_m}$

$$v_k(t,\sigma) \qquad k=1,\ldots m-1$$

$\Big\}$ $\underline{AdS_m}$

THE GRAVITATIONAL BACKGROUND

$$g_{\hat{\mu}\hat{\nu}} \qquad AdS_m \times S_n$$

$$A^{(m-1)}_{t\alpha_1\cdots\alpha_{m-2}} = \frac{\rho^{m-1}}{\tilde{L}}, \quad A^{(n-1)}_{\phi\chi_1\cdots\chi_{n-2}} = L^{n-1} r^{n-1}$$

FOR THE FLUCTUATING BRANE: INDUCED DIRAC-B-I ACTION WITH A C-S TERM

$$S = S_{DBI} + T\int P[A]$$

FINAL EXPRESSION FOR QUADRATIC FLUCT.

$$\delta S = \int d\tau\, d\vec{\sigma} \Big\{ \frac{1}{2}\frac{L^3}{L^2 - r_0^2}\dot{\eta}^2 - \frac{L^2}{2(L^2-r_0^2)} g^{\sigma_i\sigma_j}\partial_i\eta\,\partial_j\eta$$

$$+ \frac{L^3(L^2-r_0^2)}{2r_0^2}(\delta\dot{\phi})^2 - \frac{L(L^2-r_0^2)}{2r_0^2} g^{\sigma_i\sigma_j}\partial_i\delta\phi\,\partial_j\delta\phi$$

$$+ L^2\frac{(n-3)}{r_0}\eta\,\partial_\tau\delta\phi$$

$$+ \frac{L\tilde{L}^2}{2}\partial_t v_k\,\partial_t v_k - \frac{\tilde{L}^2}{2L} g^{ij}\partial_i v\,\partial_j v - \frac{L}{2}v_k v_k \Big]$$

Solution:

$$e^{i\omega t}\, Y_l(\vec{\sigma})\, \alpha_l$$

$\begin{pmatrix} \eta \\ \delta\phi \end{pmatrix}$ - coupled $\quad \omega_\pm^2 = \frac{1}{L^2}\left(Q_l + \frac{(n-3)^2}{2} \pm (n-3)\sqrt{Q_l + \frac{(n-3)^2}{4}}\right)$

v_k : $\quad \omega^2 = \frac{Q_l}{L} + \frac{1}{L^2}$

eigenfrequencies. $\quad Q_l = l(l+1)$

The excitations of the giant graviton read

$$E = J + \sum n\omega$$

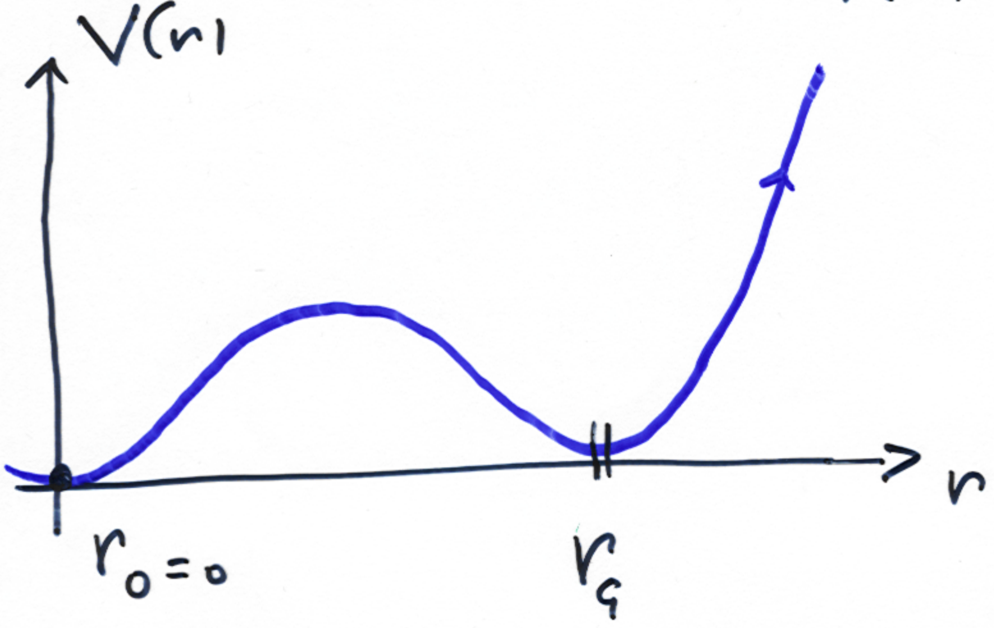
- Freq. are positive
- G. G. is stable under small fluct.

• INSTANTON SOLUTION : TUNNELING

We had an effective Hamiltonian describing radial motions

$$H = \left[\frac{p_r^2}{g_{rr}(r)} + j^2 + \frac{(jr - r^p)^2}{1-r^2} \right]^{1/2}$$

$$\frac{(jr - r^p)^2}{1-r^2} \equiv V(r)$$



Two minima

$r_0 = 0$

$r_G = j^{\frac{1}{p-1}}$

"GIANT GRAVIT"

• IS TUNNELLING BETWEEN THE $r_0 = 0$ CONF AND THE CANDIDATE GIANT GR. r_G POSSIBLE ?

- **Yes**, There is AN INSTANTON SOLUTION

DAS, A.J., S. MATHUR

hep-th/0008088

CONTINUE TO EUCLIDEAN TIME

$$S_E = \int d\tau \left[j^2 + V(r) \right]^{1/2} \left(1 + g_{rr}\, \dot{r}^2\right)$$

"SELF-DUAL" EUCLIDEAN EQUATION

$$\boxed{\frac{\partial}{\partial \tau} r(\tau) = \pm \frac{1}{\theta} r(\tau)\left(j - r^{p-1}\right)}$$

SOLUTION: EXAMPLE $p=2$ (VERY SIMPLE)

$$\frac{r}{j-r} = e^{\tau}$$

The (EUCLIDEAN) ACTION is FINITE:

$$S_{INST} = \left[j + \frac{1}{2}(1-j)\log(1-j) - \frac{1}{2}(1+j)\log(1+j) \right]$$

• NONCOMUTATIVITY (AS SEEN BY THE GIANT) G.

The GIANT GRAVITON CONFIGURATION REPRESENTS A BPS STATE WHERE 1/2 of SUSY is PRESERVED. This REDUCTION CAN BE SHOWN TO BE EQUIVALENT TO

$$\boxed{p_r = 0}$$

(time independent conf.

TAKING THIS AS A REDUCTION of phase SPACE we see the following

$\psi_1 = p_r = 0$ primary CONST.

$\psi_2 = \frac{dV}{dr} = 0$ secondary $([\psi_1, H])$

No further CONSTRAINTS AS

$$[H, \psi_2] = -\frac{p_r}{H g_{rr}} V^{(2)}_{(r)} = 0$$

by $\psi_1 = 0$

DIRAC BRACKET of r AND ϕ

$$[r, \phi]_D = [r, \phi]_P = 0$$ (This used to be commuting)

$$- [r, \psi_1] [\psi_1, \psi_2]^{-1} [\psi_2, \phi]$$

$$\rightarrow \frac{\partial^2 V / \partial r \partial j}{\partial^2 V / \partial r^2} = \frac{1}{N} \frac{r^{2-p}}{p-1}$$

We see a NON commutativity of Sphere

$$[r, \phi] = \frac{1}{N} \frac{r^{2-p}}{p-1}$$

$p=2$ This is the NON COMMUTATIVE (FUZZY) SPHERE!

SUMMARY:

W/ HAVE ~~GIVEN~~ COLLECTED EVIDENCE FOR NON COMMUTATIVITY OF $AdS_p - S_{p'}$ SPACE-TIME(S) IN A VARIETY OF CASES. The NON-COMMUTATIVITY PARAMETER IS not AN EXTERNAL FIELD BUT THE $\frac{1}{N}$ OR g_{ST} ITSELF THE CASES STUDIED ARE:

I. 2D NONCRITICAL STRING

$$[AdS_2]_2$$

II. MOST DETAILED: S_N ORBIFOLD ON T_4, K_3

$$[AdS_3]\times[S_3]_{\frac{1}{N}} \leftarrow \text{NONCOMMUTATIVE}$$

$$U(1) \downarrow \quad U(1)$$

$$[AdS]_2 \times [S_2] \quad \text{coset}$$

III. SEMICLASSICAL ARGUMENT following Susskind, McGreevy + Toumbas GIANT GRAVITON ✓