Boundary String Field Theory of the $D\bar{D}$ System

hep-th/0012198

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also: hep-th/0012210

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Introduction

- There has been a recent resurgence of interest in string field theory (SFT). Development in 80's focussed on formal and perturbative issues, and lacked a suitable non-perturbative question.

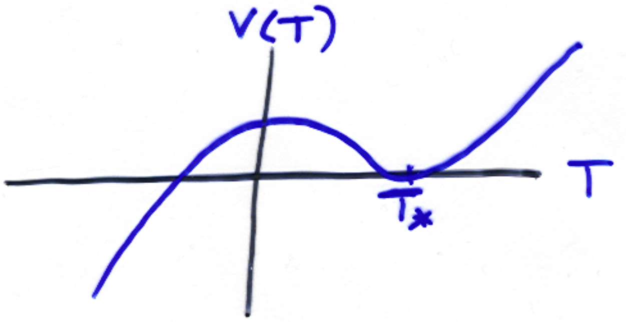
- Conjectures of Sen on open string tachyon condensation have provided a tractable non-perturbative testing ground for SFT.

- Simplest case: bosonic open string

tachyon: $\mathcal{L} \sim \frac{1}{2}(\partial T)^2 - \frac{1}{2} m_T^2 T^2 + \cdots$

$$m_T^2 = -1/\alpha'$$

Tachyon potential from SFT:



- In old days there was a mystery regarding new vacuum $T = T_*$.

New exotic phase of string theory?

- We now have a much better understanding:

open string theory $\Longrightarrow$ D25 - brane. Spacefilling "soliton"!

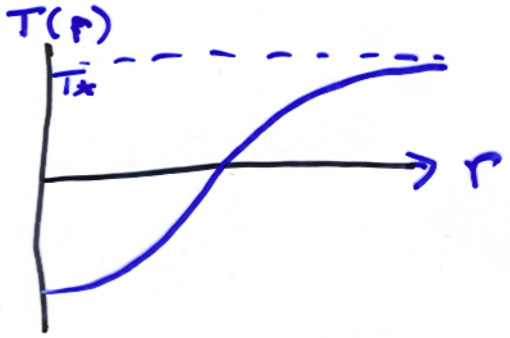
Tachyon condensation $(T \to T_*)$ represents D25-brane decaying into closed string vacuum. $(T \to -\infty$ still mysterious$)$

- Gives prediction for height of potential:

$$V(0) - V(T_*) = T_{D25}$$

One goal has been to verify this in SFT.

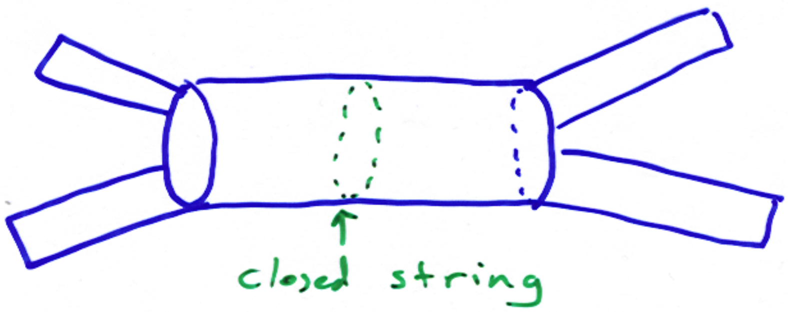
- Theory also supports soliton solutions asymptotic to T_*



We now know that such a p+1 dimensional soliton represents a Dp- brane.
SFT should reproduce tension and spectrum.

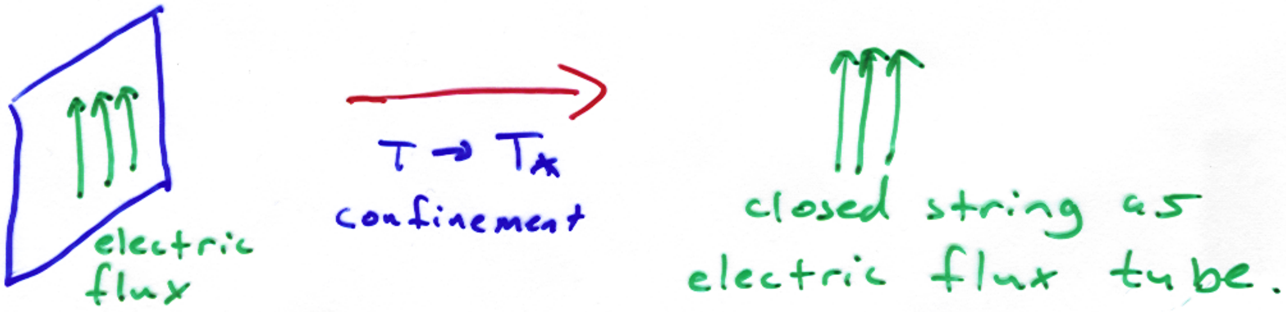
- Third — more subtle and interesting — question concerns physics around T_*.
  All open strings should have disappeared, leaving just closed strings. Can we see these closed strings in open SFT?

- At $T=0$ closed strings are well known to arise as poles in one loop open string amplitudes:



Question is whether closed strings arise differently (e.g. classically?) after tachyon condensation.

- Speculative picture:



Charge carried by $S \sim B_{\mu\nu} F^{\mu\nu}$

- Challenging to find flux tube solution.

• There have been three main approaches to these questions:

1) Level truncation of Witten's cubic SFT:

$$S = \int \psi * Q \psi + g \psi * \psi * \psi$$

$$\psi = T + A_\mu + \cdots$$

Infinite number of components condense along with T. Make tractable by keeping finite number, looking for convergence as more are kept. Good numerical evidence for height of potential and soliton solutions.

(Kostelecky, Samuel; Sen, Zwiebach; ⋮)

2) Non-commutative geometry:

Look at tachyon condensation in B-field. Theory is non-com. gauge theory. Simple construction of exact soliton solutions.

(Harvey, P.K., Larsen, Martinec; Dasgupta, Mukhi, Rajesh; Witten ⋮)

3) Boundary string field theory

Exact potential and solitons.

Witten
Gerasimov, Shatashvili
Kutasov, Marino, Moore

Outline

- Define BSFT for $D\bar{D}$ system.
- Derive various terms in action for tachyon + gauge field.
- Show that lower dimensional D-branes are solitons.
- Derive RR-couplings:

$D\bar{D}$:

$$S_{WZ} = \int C \wedge \mathrm{Str}\, e^{2\pi i \mathcal{F}}$$

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$$

$$\mathcal{A} = \begin{pmatrix} iA^+ & \bar{T} \\ T & iA^- \end{pmatrix}$$

non-BPS Dp:

$$S_{WZ} = \int C \wedge \mathrm{tr}\, e^{2\pi(iF - T^2 + DT)}$$

- Show that solitons have correct RR-charge.

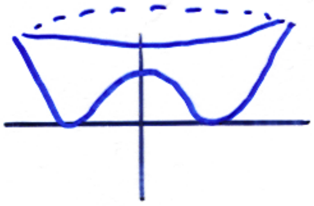
$$Q_{RR} = \mathrm{ind}(Q) = \mathrm{ind}\begin{pmatrix} i\partial\!\!\!/ + A\!\!\!/^+ & \bar{T} \\ T & i\partial\!\!\!/ + A\!\!\!/^- \end{pmatrix}$$

Review of $D\bar{D}$ system (Sen; Witten)

N $D\bar{D}$ pairs: $U(N)\times U(N)$ gauge fields

$T \sim (N,\bar{N})$ tachyon

Tachyon potential:



$\Delta V = 2T_0$

Tachyon condensation:

$$\langle T\rangle \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix} \Rightarrow U(N)\times U(N) \to U(N)$$

Vacuum manifold: $\mathcal{M} = \dfrac{U(N)\times U(N)}{U(N)}$

$$\pi_{2k-1}(\mathcal{M}) = \mathbb{Z} \Rightarrow \text{codimension } 2k \text{ solitons}$$

$\hookrightarrow$ N suff. large

So $D9-\overline{D9}$ gives $D(9-2k)$ branes of IIB

ABS construction:

$SO(2k)$ γ-matrices

$$T = f(r)\,\gamma\cdot\vec{x}$$

$= 1$ in BSFT

Non-BPS Dp: $(-)^{F_L}$: $T = \bar{T}$, $A^+ = A^-$

BSFT approach

(Witten; Shatashvili; Gerasimov, Shatashvili; Kutasov, Marino, Moore)

- Theory on space of all two dimensional field theories conformal in bulk but with arbitrary boundary interactions.

- Classical spacetime action for superstring is disk partition function:

$$S(\lambda_i) = Z(\lambda_i) = \int D\phi\, e^{-S_{bulk} - S_{bndy}}$$

$$S_{bndy} = \sum_i \lambda_i \mathcal{O}^i$$

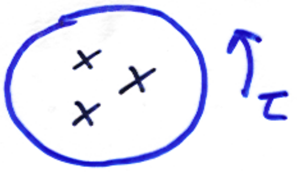
- Closely related to σ-model approach

(Fradkin, Tseytlin; Callan et. al. Andreev, Tseytlin)

⋮

Organization of computations

1) Integrate over fields in bulk with fixed boundary conditions $\phi_i(\tau)$



- We'll consider only NS-NS and RR vacua.

$$\Psi_{bulk}[\phi_i(\tau)] = \int D\phi_{bulk}\, e^{-S_{bulk}}$$

2) Include boundary interaction:

$$\Psi_{bndy}[\phi_i(\tau)] = e^{-S_{bndy}}$$

3) Integrate over boundary values:

$$Z = \int D\phi\, e^{-S_{bulk} - S_{bndy}}$$

$$= \int D\phi(\tau)\, \Psi_{bulk}\, \Psi_{bndy}$$

4) Renormalize by zeta-function

$$S = Z_{ren}$$

Bulk wavefunctional

- bulk action for NSR string: $(\alpha' = 1)$

$$S_{bulk} = \frac{1}{4\pi}\int d^2z \left(2\partial X^\mu \bar\partial X^\mu + \psi^\mu \bar\partial \psi^\mu + \tilde\psi^\mu \partial \tilde\psi^\mu\right)$$

- boundary conditions (NS sector)

$$X^\mu(\tau) = x_0^\mu + \sqrt{\tfrac{1}{2}} \sum_{n=1}^{\infty} \left(X_n^\mu e^{in\tau} + X_{-n}^\mu e^{-in\tau}\right)$$

$$\psi^\mu(\tau) = \tilde\psi^\mu(\tau) = \sum_{r=\frac{1}{2}}^{\infty} \left(\psi_r^\mu e^{ir\tau} + \psi_{-r}^\mu e^{-ir\tau}\right)$$

- S_{bulk} for solution with these boundary conditions:

$$S_{bulk} = \frac{1}{2}\sum_{n=1}^{\infty} n X_{-n}^\mu X_n^\mu + i \sum_{r=\frac{1}{2}}^{\infty} \psi_{-r}^\mu \psi_r^\mu$$

$$\Psi_{bulk} = e^{-S_{bulk}}$$

Boundary interaction

- Demand spacetime gauge invariance:

$$\delta A^{\pm}_{\mu} = \partial_\mu \alpha^{\pm} + i[\alpha^{\pm}, A^{\pm}_{\mu}]$$

$$\delta T = -iT\alpha^{+} + i\alpha^{-}T$$

and worldsheet supersymmetry

$$X^\mu = x^\mu + \theta\psi^\mu$$

$$D = \partial_\theta + \theta\partial_\tau$$

- Gauge fields and tachyons naturally packaged as

$$M(X) = \begin{pmatrix} iA^{+}_{\mu}DX^\mu & \bar{T} \\ T & iA^{-}_{\mu}DX^\mu \end{pmatrix}$$

- option 1: path ordering:

$$e^{-S_{bndy}} = \mathrm{tr}\,\hat{P}\,e^{i\int d\tau d\theta\, M(X)}$$

susy path ordering

Path ordering is awkward!

Boundary fermions

(Marcus, Sagnotti; Witten; Harvey, Kutasov, Martinec; Kutasov, Marino, Moore)

- Consider $N = 2^{m-1}$ $D9\overline{D9}$ pairs.

$$M(X) = \sum_{k=0}^{2m} \frac{1}{2k!} M^{I_1 \cdots I_k} \gamma^{I_1 \cdots I_k}$$

↑ $SO(2m)$ γ-matrices

- Introduce $2m$ boundary fermion superfields:

$$\Gamma^I = \eta^I + \theta F^I, \qquad I = 1..2m$$

$$S(\Gamma^I) = -\int d\tau\, d\theta\, \frac{1}{4} \Gamma^I D \Gamma^I$$

$$\Rightarrow \{\eta^I, \eta^J\} = 2\delta^{IJ}$$

- So replace $\gamma^I \to \Gamma^I$

$$\boxed{S_{bndy} = -\int d\tau\, d\theta \left(\frac{1}{4} \Gamma^I D \Gamma^I + \sum \frac{1}{2k!} M^{I_1 \cdots I_k} \Gamma^{I_1} \cdots \Gamma^{I_k} \right)}$$

- Manifest gauge symmetry is

$$U(1) \times SO(2m) \subset U(2^{m-1}) \times U(2^{m-1})$$

- No path ordering needed.

- Representation of matrices in terms of boundary fermions changes rule for matrix multiplication.

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \sum_k M^{I_1 \cdots I_k} \gamma^{I_1 \cdots I_k}$$

Use: $\gamma^I = \begin{pmatrix} 0 & \tilde{\gamma}^I \\ \tilde{\gamma}^I & 0 \end{pmatrix}$

with, e.g. $A = \begin{cases} \text{bosonic for} & (-)^a = +1 \\ \text{fermionic for} & (-)^a = -1 \end{cases}$

$$M \longrightarrow \sum M^{I_1 \cdots I_k} \Gamma^{I_1} \cdots \Gamma^{I_k}$$

- Multiply M, M' keeping Γ's to the right:

$$MM' = \sum M^{I_1 \cdots I_k} \Gamma^{I_1 \cdots I_k} M'^{J_1 \cdots J_{k'}} \Gamma^{J_1 \cdots J_{k'}}$$

$$= \sum (-)^{km'} M^{I_1 \cdots I_k} M'^{J_1 \cdots J_{k'}} \Gamma^{I_1 \cdots I_k} \Gamma^{J_1 \cdots J_{k'}}$$

where: $M'^{J_1 \cdots J_{k'}} = \begin{cases} \text{bosonic for} & (-)^{m'} = +1 \\ \text{fermionic for} & (-)^{m'} = -1 \end{cases}$

- In terms of matrices:

$$MM' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'} BC' & AB' + (-)^{d'} BD' \\ DC' + (-)^{a'} CA' & DD' + (-)^{b'} CB' \end{pmatrix}$$

- Will be important for getting correct tachyon covariant derivative.

- Now integrate out auxilliary fields F^I $(\Gamma^I = \eta^I + \theta F^I)$

$$S_{bdy} = -\int d\tau\, d\theta \left(\frac{1}{4} \dot{\eta}^I \eta^I + \sum \frac{1}{2k!} (M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k} \right)$$

defined by earlier matrix mult. rule

$$M = M_0 + \theta M_1$$

- Explicitly:

$$M_0 = \begin{pmatrix} i A^+_\mu \psi^\mu & \bar{T} \\ T & i A^-_\mu \psi^\mu \end{pmatrix}$$

$$M_1 = \begin{pmatrix} i\left(A^+_\mu \dot{x}^\mu + \frac{1}{2}(dA^+)_{\mu\nu} \psi^\mu \psi^\nu\right) & \partial_\mu \bar{T} \psi^\mu \\ \partial_\mu T \psi^\mu & i\left(A^-_\mu \dot{x}^\mu + \frac{1}{2}(dA^-)_{\mu\nu} \psi^\mu \psi^\nu\right) \end{pmatrix}$$

- $M_0{}^2$ terms give $[A_\mu, A_\nu]$ in non-abelian field strength as well as well as correct non-abelian tachyon covariant derivative

Examples

- Take single $D9\,\overline{D9}$ pair $(m=1)$

$$S_{bndy} = -\int d\tau \Big(-\tfrac{1}{4} T^I T^I + \tfrac{1}{4} \dot{\eta}^I \eta^I + \tfrac{1}{2} D_\mu T^I \psi^\mu \eta^I$$
$$+ \tfrac{i}{2} \left(\dot{X}^\mu A_\mu + \tfrac{1}{2} F_{\mu\nu} \psi^\mu \psi^\nu \right)$$
$$+ \tfrac{i}{4} \left(\dot{X}^\mu A_\mu^{IJ} + \tfrac{1}{2} F_{\mu\nu}^{IJ} \psi^\mu \psi^\nu \right) \eta^I \eta^J \Big)$$

$(I, J = 1, 2)$

- T charged with respect to relative gauge field A_μ^{IJ}.

- Interacting theory, so we can't work out partition function in closed form. Consider special cases:

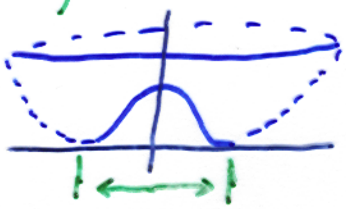
1) $T^I =$ constant, $F_{\mu\nu} =$ constant, $F_{\mu\nu}^{IJ} = 0$
   - tachyon and gauge fields decouple.

$$S = Z = 2 T_{D9} \int d^{10}x \, e^{-2\pi T \overline{T}} \sqrt{\det(\eta_{\mu\nu} + 2\pi F_{\mu\nu})}$$

(Sen)

$T = \frac{1}{2}(T^1 + i T^2)$

- $V(T, \overline{T}) = 2 T_{D9} \, e^{-2\pi T \overline{T}}$



2) T^I, $F_{\mu\nu}$, $F^{IJ}_{\mu\nu}$ = constant

- Work out partition function perturbatively. To second order in gauge fields:

$$S = 2T_{D9}\int d^{10}x\, e^{-2\pi T\bar{T}}\Big(1 + 8\pi\ln(2)\, D_\mu T D^\mu \bar{T}$$

$$+ \frac{(2\pi)^2}{8}(F^+_{\mu\nu})^2 + \frac{(2\pi)^2}{8}(F^-_{\mu\nu})^2 + \frac{\beta}{8} T\bar{T}(F^+_{\mu\nu} - F^-_{\mu\nu})^2\Big)$$

- Higher orders: action multiplied by overall factor of $e^{-2\pi T\bar{T}}$, so vanishes in closed string vacuum $T\bar{T}\to\infty$

3) Linear tachyon profiles, $A_\mu = A_\mu^{IJ} = 0$

- Relevant for describing D-branes as solitons

(Kutasov, Marino, Moore)

- By spacetime and gauge rotations take

$$T^I = u^I x^I, \qquad I = 1, 2$$

- Path integral is Gaussian and yields:

$$S = \int \frac{d^{10}x}{(2\pi)^5} e^{-2\pi T\bar{T}} \prod_{I=1}^{2} F(\pi y^I)$$

$y^I = (u^I)^2$

$$F(x) = \sqrt{2\pi}\, \frac{\prod_{r=1/2}^{\infty} (1 + x/r)}{\prod_{n=1}^{\infty} (n + x)} \underset{\text{zeta}}{=} \frac{4^x \cdot x \cdot \Gamma(x)^2}{2\Gamma(2x)}$$

- Stationary point correspond to $y^I = 0$ or ∞.

a) $y^1 = \infty$, $y^2 = 0$, $\quad T = y^1 x^1$

- describes kink $\Longrightarrow$ non-BPS D8-brane (Sen)

$$S = 2T_{D9} \int d^{10}x \, e^{-\frac{\pi}{2} y^1 (x^1)^2} F(\pi y^1) \Big|_{y^1 = \infty}$$

$$= 2\pi\sqrt{2}\, T_{D9} \int d^9x \Longrightarrow T_{\text{non-BPS } D8} = 2\pi \sqrt{2}\, T_{D9} \checkmark$$

b) $y^1 = y^2 = \infty$, $\quad T = y^1 x^1 + i y^2 x^2$

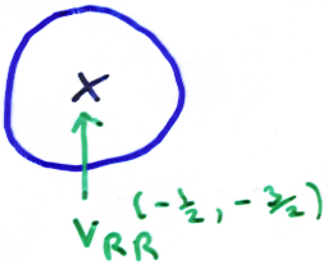
- describes vortex $\Longrightarrow$ BPS D7-brane (Sen)

RR couplings

- Turn on constant C field and find coupling generalizing

$$\int C \wedge e^{2\pi F}$$

- Bulk wavefunctional now obtained from



$$V_{RR}^{(-\frac{1}{2},-\frac{3}{2})} = S^a C_{ab} S^b e^{-\frac{1}{2}\phi(0)} e^{-\frac{3}{2}\tilde{\phi}(0)}$$

- Fermions become integer moded, and $C_{\mu_0 \cdots \mu_p}$ label zero mode wavefunctions:

$$\Psi^{RR}_{bulk} = \exp\left[-\frac{1}{2}\sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_n - i\sum_{n=1}^{\infty} \psi^\mu_{-n}\psi^\mu_n\right]$$

$$\cdot \sum_{p\ \text{odd}} \frac{(-i)^{(9-p)/2}}{(p+1)!} C_{\mu_0 \cdots \mu_p} \psi_0^{\mu_0} \cdots \psi_0^{\mu_p}$$

- Want to compute

$$Z_{RR} = \int DX\, D\psi\, D\eta\, e^{-S_{bndy}} \Psi^{RR}_{bulk}$$

$$S_{bndy} = -\int d\tau \left(\frac{1}{4}\dot{\eta}^I \eta^I + \sum \frac{1}{2k!}(M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k}\right)$$

- Focus on particular $C_{01\ldots p}$. Let all fields be independent of $X^0 \ldots X^p$. Partition function factorizes:

$$Z_{RR} = \left[\int \prod_{\mu=0}^{p} DX^\mu D\psi^\mu (-i)^{\frac{q-p}{2}} C_{0\ldots p} \psi_0^0 \cdots \psi_0^p e^{-S^{(1)}_{bulk}}\right]$$

$$\cdot \left[\int \prod_{\mu=p+1}^{q} DX^\mu D\psi^\mu D\eta \, e^{-S^{(2)}_{bulk} - S_{bndy}}\right]$$

- First factor easily computed.

Second is Witten index $\Rightarrow$ nonzero modes cancel by susy.

- Restricting to zero modes: $S_{bulk} = 0$,

$$M_0 = \begin{pmatrix} iA^+ & \bar{T} \\ T & iA^- \end{pmatrix} \equiv i\mathcal{A} = \text{superconnection}$$

(Quillen)

$$M_1 = \begin{pmatrix} idA^+ & d\bar{T} \\ dT & idA^- \end{pmatrix} \equiv id\mathcal{A}$$

where, e.g. $dT = \partial_\mu T \psi_0^\mu$

- Combination appearing in action is curvature of $\mathcal{A}$

$$M_1 - M_0^2 = i(d\mathcal{A} - i\mathcal{A}\wedge\mathcal{A}) = i\mathcal{F} = \begin{pmatrix} iF^+ - T\bar{T} & D\bar{T} \\ DT & iF^- - \bar{T}T \end{pmatrix}$$

- Integrating over periodic η^I gives

$$Z_{RR} = T_{Dq} \int C \wedge \operatorname{Str} e^{2\pi i \mathcal{F}}$$

(conjectured by Kennedy, Wilkins)

$\operatorname{Str} M = \operatorname{tr}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M$

- Can now compute RR charge of solitons.

ABS: $\gamma^i = SO(2m)$ γ-matrices

$$\gamma^{i=1\ldots 2m-1} = \begin{pmatrix} 0 & \tilde{\gamma}^i \\ \tilde{\gamma}^i & 0 \end{pmatrix}, \quad \gamma^{2m} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Tachyon profile: $\begin{pmatrix} 0 & \bar{T} \\ T & 0 \end{pmatrix} = u \gamma^i x^i$

- Nonvanishing RR coupling is:

$$S = T_{D9} \int C \wedge \frac{1}{(2m)!} e^{-2\pi u^2 \vec{x}^2} \mathrm{Str} (2\pi u \gamma^i dx^i)^{2m}$$

$$= T_{D(9-2m)} \int C_{10-2m} \quad \checkmark$$

RR coupling for non-BPS D-brane

$(-)^{F_L}$ orbifold: $T = \bar{T}$, $A^+ = A^-$, omit η^{2m}
(Sen)

$$S = \frac{T_{D9}}{\sqrt{2}} \int C \wedge \mathrm{Str}\, e^{2\pi i \mathcal{F}}$$

$$\mathcal{F} = \begin{pmatrix} iF - T^2 & DT \\ DT & iF - T^2 \end{pmatrix}, \quad \mathrm{Str} M = \mathrm{tr} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M$$

or $$S = \sqrt{2}\, T_{D9} \int C \wedge \mathrm{tr}\, e^{2\pi (iF - T^2 + DT)}$$

- dT term computed by (Billo, Craps, Roose; Kutasov, Marino, Moore)

RR charge as index

- S_{WZ} came from path integral in susy QM with periodic boundary conditions.

$$Z = \int_{\text{per. b.c.}} D\phi\, e^{-S} = \mathrm{Tr}(-)^F e^{-\beta H} = \mathrm{ind}(Q)$$

- Find index theorem by writing operator form of Q

(Witten; Alvarez-Gaumé; Friedan, Windey)

- Use action:

$$S = \frac{1}{4}\int d\tau\, d\theta\, DX^\mu D^2 X^\mu$$

$$-\int d\tau \left(\frac{1}{4}\dot{\eta}^I \eta^I + \sum \frac{1}{2k!}(M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k} \right)$$

- Susy transformations:

$$\delta X^\mu = \epsilon \psi^\mu$$

$$\delta \psi^\mu = \epsilon \dot{X}^\mu$$

$$\delta \eta^{I_1} = F^{I_1} = \sum_{k=1}^{2m} \frac{(-)^k}{(k-1)!} M_0^{I_1 \cdots I_k} \eta^{I_2} \cdots \eta^{I_k}$$

auxilliary field

- Canonical quantization

$$[X^\mu, P_\nu] = i\delta^\mu{}_\nu$$

$$\{\psi^\mu, \psi^\nu\} = -2\delta^{\mu\nu}$$

$$\{\eta^I, \eta^J\} = 2\delta^{IJ}$$

- Supercharge works out to be

$$Q = i\psi^\mu p_\mu - \sum \frac{1}{2k!} M_0{}^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k}$$

$$= i\psi^\mu p_\mu - i\mathcal{A}$$

- Represent commutation relations by

$$p_\mu \to -i\partial_\mu, \quad \psi^\mu \to i\gamma^\mu, \quad \eta^I \to \gamma^I$$

$$\Longrightarrow \quad Q = \begin{pmatrix} i\not{\partial} + \not{A}^+ & \overline{T} \\ T & i\not{\partial} + \not{A}^- \end{pmatrix}$$

- Keeping track of zero mode norm., index theorem is:

$$\mathrm{ind}\begin{pmatrix} i\not{\partial} + \not{A}^+ & \overline{T} \\ T & i\not{\partial} + \not{A}^- \end{pmatrix} = \left(\frac{-i}{4\pi^2}\right)^n \int \mathrm{Str}\, e^{2\pi i \mathcal{F}}$$

2n dim. manifold

- Index counts zero eigenvalues weighted by

$$(-)^F = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

- RR charge of solitons given by index.

Conclusion

- Three approaches to constructing D-branes as solitons

1) Level truncation
   - violates gauge invariance
   - involves infinite number of fields

2) NC geometry
   - Solitons have gauge field which sets covariant derivatives to zero (finite B).

3) BSFT
   - Solitons have vanishing gauge field

(1) vs. (2), (3): Need transformation involving full string field.

(2) vs. (3): Generalization of Seiberg-Witten map which can take solutions with $A_\mu = 0$ to solutions with $A_\mu \neq 0$.