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Co4

Branes cycles & deformed instanton equations

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plan:

* Introduction

  D-brane actions, κ-symmetry & Γ-operator

  Geometric D-branes

  "S-W limit" and relation to NC YM

* BPS configurations deformed conditions

  calibrations (case by case study)

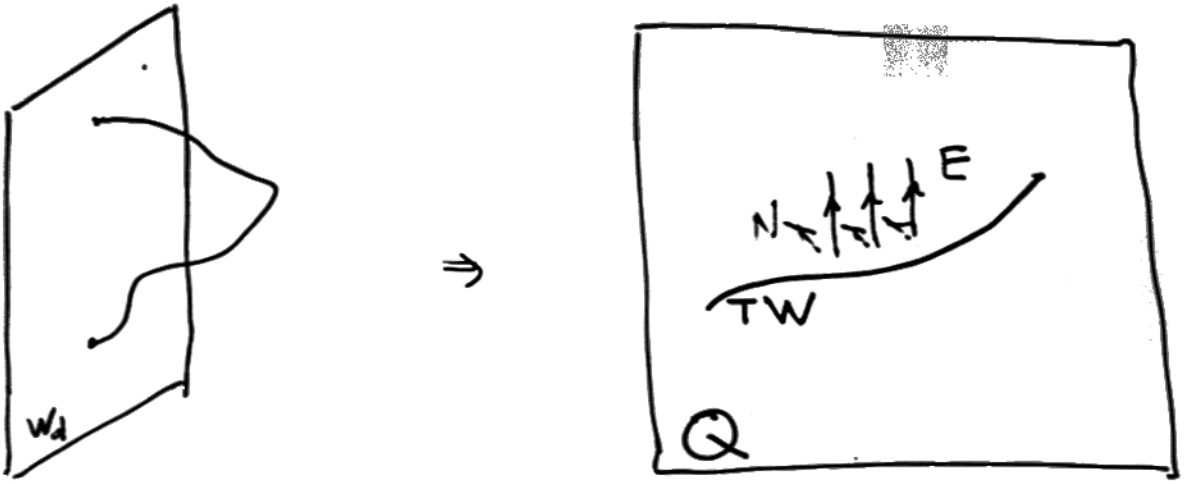
  general features

* Group-theoretic basis for deformed instantons

* Relation to NC instantons

* Other nonlinear deformations of instanton equations

* M-theory cycles



$N := N(W \hookrightarrow Q)$

E Chan-Paton bundle

$$I_p = I_{DBI} + I_{WZ}$$
$$= -T_p \int_{W_d} d^{p+1}\sigma \; e^{-\phi} \sqrt{\det(g+M)}$$
$$+ \mu \int_W C \wedge \underbrace{e^M}_{ch(E)} \left(\sqrt{\frac{\hat{A}(TW)}{\hat{A}(N)}} \, e^{-\frac{1}{2} c_1(N)} \right)$$

$M = 2\pi\alpha'(F+B)$

$C = \sum_{r=0}^{10} C^{(r)}$ RR potentials

SUSY + κ-symmetry $\rightarrow$ for BPS configurations

$$(1-\Gamma)\eta = 0$$

$\operatorname{tr}\Gamma = 0$

$\Gamma^2 = 1$

$$\frac{\sqrt{\det g}}{\sqrt{\det(g+M)}} \sum \frac{}{2^n n!} \gamma^{\mu_1 \nu_1 \quad \mu_n \nu_n} M_{\mu_1 \nu_1} \quad M_{\mu_n \nu_n} J^{(n)}_{(p)}$$

$$J^{(n)}_{(p)} \begin{cases} \Gamma_{1} \quad {}^{\frac{p-2}{2}} \quad \Gamma_{(0)} & \mathrm{II}a) \\ (-1) \quad \sigma_3 \quad {}^{\frac{p-3}{2}} \quad \sigma_2 \otimes \Gamma_{(0)} & (\mathrm{II}b) \end{cases}$$

$$\Gamma_{(0)} \qquad (p+1)! \sqrt{|g|} \quad \epsilon^{\mu_1 \quad \mu_{p+1}} \gamma_{\mu_1 \quad \mu_{p+1}}$$

$$(*) \qquad = \frac{\sqrt{|g|}}{\sqrt{|g+M|}} \propto \quad \Gamma_{(0)} \quad +$$

$$(**) \qquad \boxed{\Gamma = e^{-a/2} \, \Gamma'_{(0)} \, e^{+a/2}}$$

$$\Gamma_{(0)} \qquad \mathrm{tr}\, \Gamma'_{(0)} \qquad (\Gamma \ )^2$$

$$a \quad a(M) = \begin{cases} \frac{1}{2} Y_{ij} \gamma^{ij} \Gamma_{11} & (\mathrm{II}a) \\ \frac{1}{2} \gamma^{ij} Y_{jk} \otimes \sigma_3 & (\mathrm{II}b) \end{cases}$$

$$Y = \frac{1}{2} Y_{ij} \, e \quad e^j \quad \sum^{[p+1/2]} \phi_{2r-1} \quad e^{2r-1} \wedge e^{2r}$$

$$M \quad \frac{1}{2} M_{ij} \, e \wedge e^j \quad \sum_{r=0}^{[p+1/2]} \left(\tan \phi_{2r-1,\, 2r} \right) e^{2r-1} \wedge e^{2r}$$

In any orthonormal frame components

$$\underline{M = \tanh Y}$$

Susy cycles D branes (in geometrical phase)

D brane (susy cycle

embedded manifold

* the base
* the normal bundle

gauge connection

C.P bundle

The base M O

Solve ($\Gamma_{(0)}$ η O

for manifolds of irreducible trivial holonomy

p+	SU(2	SU(3)	G	SU(4	Spin(7
2	divisor/SL	holomorphic	–	holomorphic	
		SL	associative		
	X	divisor	ass	Cayley	Cayley
5					
		X		divisor	
7			X		
8			–	X	X

The normal bundles deformations moduli spa

accordance th susy

Ch P bundle ? α 0 reduce to YM theory

$\delta\lambda$ $\gamma^{\mu\nu} F_{\mu\nu}$

S W limi a NC instanton equations

DBI $\xrightarrow{\hspace{2cm}}$ NC YM

explicit change of variables

$[\quad x^j \quad \theta$

$(x) \quad g(x) \quad f_S \qquad \theta^{ij}\partial_i f\, \partial_j$

$\hat{F} \quad d\hat{A} \quad A \quad \hat{A}$

$\delta_\lambda \hat{A} \quad \partial\lambda \quad \lambda * A$

SW limit

$\epsilon\,\delta$

$\epsilon \to 0$

$\epsilon^{1/2} \quad 0$

$\theta^{ij} \quad \left(\frac{1}{B}\right)^{ij} \quad ij$

0

open string metric $\quad G^{ij} \quad \frac{1}{(2\pi\alpha')}\left(\frac{1}{B}\, g\, \frac{1}{B}\right)$

g_{ij}

Susy of DBI

linear $\quad \delta\lambda \quad \frac{1}{2\pi\alpha} M_{ij}^{+} \sigma^{ij} \quad \eta_\beta$

$\bar{\lambda} \quad \frac{1}{2\pi\alpha} M_{ij}\, \sigma^{ij} \quad \bar{\eta}$

non linear $\quad {}^*\lambda \quad \frac{\epsilon}{4\pi\alpha'}\eta_\alpha \qquad \delta^*\bar{\lambda} \quad \epsilon$

$\downarrow$ go to DBI

$\delta^*\lambda_\alpha \quad \frac{1}{4\pi\alpha'}\left[\epsilon \quad \mathrm{Pf}\, M \quad \sqrt{|\epsilon\delta + M|}\ \right]$

For BPS states. $\quad M^{+} \quad B^{+}$

B const $\quad 2\pi\alpha \;\; \mathrm{Pf}\, B \quad \mathrm{Pf}\, M$

or $$F^+ = \frac{B^+}{8\,\mathrm{Pf}B}\,\epsilon^{ijkl}\left(2B_{ij}F_{kl} + F_{ij}F_{kl}\right) \qquad (\star)$$

to open string frame using
$$E = -\frac{\sqrt{\epsilon}}{2\pi\alpha'}\,\frac{1}{B}$$

$$F_G^+ = (E^t)^{-1}\left(E^t F E\right)^+ E^{-1}$$
first map to orthonormal frame
then take self-duality

after doing so
$$(\star) \rightarrow \quad F_G^+ = -\frac{1}{8\,\mathrm{Pf}B}\,B_G^+\,\epsilon^{ijkl}F_{ij}F_{kl}$$

is the same as $\hat{F}^+ = 0$

$$\hat{F}(\theta=0) = F \qquad \hat{F} = \frac{1}{1+F\theta}F = B\,\frac{1}{B+F}\,F$$

$$\hat{F}_{ij}^+ = F_{ij}^+ + \tfrac{1}{2}\left(\theta^{kl}\{F_{ik}, F_{jl}\}\right)^+ + O(\theta^2)$$
$$\rightarrow F_{ij}^+ \quad \tfrac{1}{4}(F\tilde{F})\,\theta_{ij}^+ \quad)$$

BPS configurations in susy DBI $\rightleftarrows$ NC instantons

* amount of susy?
* different geometries?
* various dimensions?

BPS configurations for D-branes

Solve $\Gamma \quad \eta \quad 0$

SU(2) holonomy

$SL(2,\mathbb{R})$ doublet $\qquad \gamma_{\bar{m}}\eta \qquad\qquad \eta_{\pm} \leftarrow$ c.c.

$\gamma_{mn}\eta \quad \Omega_{mn}\eta$

$P = \quad \dfrac{\sqrt{|g|}}{\sqrt{|g+M|}} \quad \pm\gamma^{\mu\nu}M_{\mu\nu}) \Gamma_{(0)} \eta \qquad \eta_2$

$\dfrac{\sqrt{|g|}}{\sqrt{|g+M|}} \; (\quad \gamma^{\mu\nu}M_{\mu\nu}) \Gamma_{\cdot} \; \eta_2 \qquad \eta$

$\Rightarrow \qquad \left(\begin{matrix} (J) & M \\ & f^*(J) \end{matrix} \right) \quad U \left(1 \quad \dfrac{\sqrt{|g+M|}}{\sqrt{|g|}} \; Vol_2 \right.$

$U(2) \qquad$ tr.x

K3 $\to$ S of complex structures

$f^*(J) \quad Re(f^*\Omega \; , \; Im \; f^*\Omega)) \qquad$ sphere of radius Vol_2

choose J $\to$ choose a direction (holomorphic condition on cycle

normal plane $\cap S^2 \qquad S \to$ family of SLAG

in 2-d $\qquad M \qquad |g|+M \quad \to$

$\times \qquad$ fill $\qquad$ sphere of radius Vol_2

$\Rightarrow$ M does not change the calibrati condition

$p=3$

$$\begin{cases} J \wedge M & k\,(\mathrm{vol}_4 \quad \tfrac{1}{2} M \wedge M) \\ M^{2,} & 0 \end{cases}$$

following from

$$\begin{pmatrix} \frac{1}{2}(J & M) \\ M & \Omega \end{pmatrix} \quad U \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \frac{\sqrt{|g+M|}}{\sqrt{|g|}}\, \mathrm{Vol}_4$$

SU(3) holonomy

$p=1$

$$f\,(J. + M \qquad e^{i\theta}\, \frac{\sqrt{|g+M|}}{\sqrt{|g|}}\, \mathrm{Vol}_2 \qquad (a$$

$$dX \wedge dX \quad \Omega_{mnp} \quad 0 \qquad (b$$

(b) $\rightarrow$ holomorphy

$\rightarrow$ cycle undeformed

$\rightarrow$ $f^*J \quad \mathrm{vol}_2$

$\tan\theta \quad \mathrm{Vol}_2$

fix $\int_{\Sigma_2} F \quad 2\pi n \qquad \rightarrow \qquad 2\pi n + \int f\,(B) \quad \frac{\tan\theta}{2\pi\alpha'} \int J$

$p=2$

$$f^*(\Omega \qquad e^{i\theta}\, \frac{\sqrt{|g+M|}}{\sqrt{|g|}}\, \mathrm{Vol}_3$$

$$f^*(J + M \quad 0$$

$f^*(J)$ is real $\Rightarrow$ $M \quad 0$

$\Rightarrow$ $SLAG_3$ undeformed

M2 undeformed explains this

$\underline{p=3}$

$$\frac{1}{2}\left(f^*(J)+iM\right)^2 = e^{i\theta}\frac{\sqrt{|g+M|}}{\sqrt{|g|}}\,\mathrm{Vol}_4$$

$$f^*(\Omega)\wedge dX^{\bar q}g_{\bar q r} + M\wedge dX^m\wedge dX^n\,\Omega_{mnr} = 0$$

consider explicitly an embedding $X^3 = X^3(X^1,\bar X^1, X^2, \bar X^2)$

$\rightarrow$ can show it is still holomorphic & learn

$$M^{2,0}=0$$

for (1,1) part

$$f^*(J)\wedge M = \tan\theta\left(\mathrm{vol}_4 - \frac{1}{2}M\wedge M\right)$$

(Usual instanton : $M^{2,0}=0$

$f^*(J)\wedge M = k\,\mathrm{vol}_4$)

$\underline{p=5}$

$$\frac{1}{2!}J\wedge J\wedge M - \frac{1}{3!}M\wedge M\wedge M = \tan\theta\left(\mathrm{vol}_6 - \frac{1}{2!}J\wedge M\wedge M\right)$$

$$M^{2,0}=0$$

$\underline{G_2 \text{ holonomy}}$ for c.c. θ $\quad \gamma_{\mu\nu\rho}\theta = \Phi_{\mu\nu\rho}\theta$

$\underline{p=2}$

$$f^*(\Phi) = \frac{\sqrt{|g+M|}}{\sqrt{|g|}}\,\mathrm{Vol}_3$$

$$M\wedge dX^\mu = 0$$

$\Rightarrow M=0$ no deformations!

$\underline{p=3}$

$$f^*(*\Phi) - \frac{1}{2}M\wedge M = \frac{\sqrt{|g+M|}}{\sqrt{|g|}}\,\mathrm{Vol}_4$$

$$M\wedge dX^\mu\wedge dX^\nu\,\Phi_{\mu\nu\rho} = 0$$

can show $\quad M^+ = 0$

SU(4) holonomy

$\rightarrow$ holomorphic cycle

$$\frac{1}{2}(f^*(J)+iM)^2+f(\bar{\Omega}_0)=e^{i(\phi+\theta)/2}\frac{\sqrt{|g+M|}}{\sqrt{|g|}}\,\mathrm{Vol}_4$$

$$\mathrm{Im}\left(e^{i(\theta+\phi)/2}f^*(\Omega_0)\right)=0$$

$$(f^*(J)+iM)\wedge\left(dX^\mu\wedge dX^\rho+\frac{1}{2}(\bar{\Omega}_0)_{\bar{\rho}\bar{\sigma}}{}^{\mu\rho}\,dX^{\bar{\rho}}dX^{\bar{\sigma}}\right)=0$$

when $M=0$ $\quad\theta+\phi=0\quad\rightarrow$ Cayley calibration

$$-\frac{1}{2}(f^*(J))^2+\mathrm{Re}(f^*(\Omega))=0$$

when $M=0$ $\rightarrow$ holomorphic embedding $\rightarrow$ divisor

Spin (7) holonomy

$$f^*(\Omega)-\frac{1}{2}M\wedge M=\frac{\sqrt{|g+M|}}{\sqrt{|g|}}\,\mathrm{Vol}_4$$

$$P(M\wedge dX^\mu\wedge dX^\nu)=0 \qquad (\Omega=\Phi\wedge dX^?+*\Phi)$$

$\mathrm{Spin}(8)\rightarrow\mathrm{Spin}(7)\qquad 28\rightarrow 21+7$

P: projector $28\rightarrow 7$

again can show $\underline{M^+=0}$ $\quad(M_{\nu\rho}=\Omega_{\nu\rho}{}^{\mu\lambda}M_{\mu\lambda})$

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general comments:

$\rightarrow$ Vol $\dfrac{\sqrt{|g+M|}}{\sqrt{|g|}}$

$f^*(J)\rightarrow f^*(J)+iM$ $\leftarrow$ complexified Kähler form

$P_-^2=P_-\rightarrow P_-\eta=0\rightarrow\eta^+P_-\eta=0$

$\Rightarrow$ top-form equation $\rightarrow$ the main equation

Group-theoretic basis $\qquad$ turn to $\mathbb{R}^{p+1} \times M_{q-p}$

Recall $\qquad \Gamma = e^{-a/2}\, \Gamma'_{(0)}\, e^{a/2}$

$$a \sim \tfrac{1}{2} Y^{ij} \gamma_{ij}$$

find χ: $\qquad \tfrac{1}{2} Y_{ij} \gamma^{ij} \chi = k \chi \qquad (*)$

Can solve $(1-\Gamma)\eta = 0$ by $\begin{cases} \eta_1 = \chi \\ \eta_2 = \pm i^{\frac{p+3}{2}} \chi \end{cases}$

$$\underline{M = \tanh Y} \longrightarrow \text{deforming instantons}$$

$dM = 0 \;\rightarrow$ complicated constraint on Y

$\left(\mathrm{spin}(p+1) = h_{\parallel} \oplus h_{\perp} ;\ (*) \Rightarrow \begin{cases} Y_{\perp} = 0 \\ Y_{\parallel} = \text{const!} \end{cases} \overset{?}{\Rightarrow} M \right)$

$\underline{p=3}$ $\qquad Y = (y_{12} T_{12} + y_{34} T_{34})$

$$T_{ij} = e_{ij} - e_{ji}$$

$$Y^{+} = \tfrac{1}{2}(y_{12} + y_{34})(T_{12} + T_{34})$$

$\tanh Y = \tanh y_{12}\, T_{12} + \tanh y_{34}\, T_{34}$

use addition formula for tanh $\Rightarrow$

$$(\tanh Y)^{+} = \tfrac{1}{2} \tanh(2Y^{+})\,(1 - \mathrm{Pf}(\tanh Y))$$

solve $\qquad -Y^{ij}\gamma_{ij}\, \eta = k\eta$

and use that for $g_{ij} = \overset{\text{SWL}}{\epsilon}\, \delta_{ij}$, $\underline{(M = \epsilon \tanh Y)}$

$\Rightarrow Y^{+} = \text{const} \qquad$ define $\xi = \epsilon / 2\pi\alpha'$

$$\frac{\xi^{-1}(F+B)^{+}}{1 - \xi^{-2}\, \mathrm{Pf}(F+B)} = \tfrac{1}{2} \tanh(2Y^{+}) = \text{const}$$

evaluate the constant at ∞ $\quad F \to 0$
$B = \text{const}$

$$\Rightarrow \quad \frac{(F+B)^+}{Pf(F+B) - \xi^2} = \frac{B^+}{Pf B - \xi^2}$$

<u>$p = 5, 7$</u>

on $\mathbb{R}^6$:

$$\begin{cases} Y^{2,0} = 0 \\ J^{m\bar{n}} Y_{m\bar{n}} = k \end{cases} \qquad (+)$$

use $\quad M/\epsilon = \tanh Y$

Take $\quad J = \{ T_{12} + T_{34} + T_{56} \} \quad \Rightarrow \quad U(3) \subset SO(6)$

$$15 = 1 \oplus 3 \oplus 3^* \oplus 8$$

$$Y = Y_1 + Y_3 + Y_{3^*} + Y_8$$

$(+) \Rightarrow \quad Y_3 = Y_{3^*} = 0 \qquad Y_1 = \text{const}\ \mathbb{1}$

$\tanh Y$ is a <u>power series</u> $\Rightarrow \quad Y_3 = 0 \Rightarrow (\tanh Y)_3 = 0$

$$Y^{2,0} = 0 \quad \Rightarrow \quad M^{2,0} = 0$$

if $Y_3 = Y_{3^*} = 0$ can prove

$$\frac{(\tanh Y)_1 - \frac{1}{3} Pf(\tanh Y) J}{1 - \frac{1}{2} Pf(\tanh Y)\, Tr(J(\tanh Y)^{-1})} = \frac{1}{3}\tanh(3Y_1)$$

$\uparrow$

l.h.s - nonlinear $f(M/\epsilon)$

evaluate const at ∞

$$\frac{\xi^2(\mathcal{F}\wedge\omega^2)/2! - \mathcal{F}^3/3!}{\xi^2\omega^3/3! - (\mathcal{F}^2\wedge\omega)/2!} = \frac{\xi^2(B\wedge\omega^2)/2! - B^3/3!}{\xi^2\omega^3/3! - (B^2\wedge\omega)/2!}$$

$\omega = f^*(J)$

Similarly for $p=7$ using $28 = 1 + 6 + 6^* + 15$ $(U(4) \subset SO(8)$

Can be put together in form:

$$\frac{[e^{\omega} \, \mathrm{Sin}(\zeta^{-1}(F+B))]^{top}}{[e^{\omega} \, \mathrm{Cos}(\zeta^{-1}(F+B)]^{top}} = \frac{[e^{\omega} \, \mathrm{Sin}(\zeta^{-1} B)]^{top}}{[e^{\omega} \, \mathrm{Cos}(\zeta^{-1} B)]^{top}}$$

+ G_2 $\quad Y = Y_7 + Y_{14}$ $\quad$ get $Y_7 = 0 \Rightarrow (\tanh Y)_7 = 0$

$\Rightarrow$ undeformed instantons obtained in $\alpha' = 0$ limit

$F_{\mu\nu} = (* \Phi)_{\mu\nu}{}^{\rho\lambda} F_{\rho\lambda}$

Spin(7) $\quad Y = Y_7 + Y_{21}$ $\rightarrow$ similar conclusions

Relations to NC instantons

$\zeta \to 0$ limit $\qquad \dfrac{(F+B)^+}{Pf(F+B)} = \dfrac{B^+}{Pf\, B}$ $\qquad$ in $\underline{p=3}$

$$\Rightarrow \quad \underline{\hat{F}^+ = 0}$$

$\underline{p=5,7}$ $\qquad \zeta \to \infty \quad \Rightarrow$ ordinary Hermitian YM

$$\begin{cases} F^{2,0} = 0 \\ F \wedge \omega^{\frac{p-1}{2}} = 0 \end{cases}$$

SWL $\zeta \to 0$

$$\begin{cases} \dfrac{(F+B)^{\frac{p-1}{2}} \wedge \omega}{Pf\,(F+B)} = \dfrac{B^{\frac{p-1}{2}} \wedge \omega}{Pf\,(B)} \\ (F+B)^{2,0} = 0 \end{cases}$$

(1, 1) forms

$$\phi^{\frac{p+1}{2}} = \mathrm{Pf}(\phi)\, \omega^{\frac{p+1}{2}}$$

$$\phi^{\frac{p-1}{2}} \wedge \omega = \mathrm{Pf}(\phi)\, \mathrm{Tr} \quad J \phi \quad)$$

$$\Rightarrow \quad \mathrm{Tr}\left[J \left({}_{F+B} \qquad \frac{1}{B} \right) \right] \quad 0$$

but $\quad F = \frac{1}{1+F\theta} F \qquad B \left(\frac{1}{B+F} \quad \frac{1}{B} \right) B$

using $\quad E \quad 2\pi\alpha^{1/2} \quad /B$

$$\Rightarrow \quad \mathrm{Tr}\left(J_q F \right) \quad 0$$

since $\quad B \qquad (1 \ 1)$

$$\begin{cases} \hat{F}^{2,} \qquad 0 \\ F \quad \omega^{\frac{p-1}{2}} = 0 \end{cases}$$

$F^{(2,0)} \quad 0$ eom for chiral cocycle theories

study (1,1) connections on holo phic bundles governed by actions formed from Bott Chern classes)

↑

admit <u>non-abelian</u> generalizat ons

MS & Kodaira - Spencer theory

$$\Gamma = \mp i \Gamma_0 \left(1 - \frac{1}{2 \cdot 3!} h_{\mu\nu\rho} \gamma^{\mu\nu\rho} \right)$$

$$g^{\mu\rho} \left(\delta_\mu{}^\nu - 2 h_{\mu\rho'\lambda'} h^{\nu\rho'\lambda'} \right) \partial_\mu h_{\nu\kappa\beta} = 0$$

$dH = 0$ $\quad$ $*h = -ih$ $\quad$ nonlinear condition on $H = d\beta_2 + \ldots$

Solve $\quad (1 - \Gamma)\epsilon = 0$

self-duality of h:

$$h = h^{3,0} + h^{2,1} + h^{1,2}$$

Im$(J \wedge \ldots)$ $\qquad$ Ker$(J \wedge \ldots)$

$$(1-\Gamma)\epsilon = 0 \Rightarrow \begin{cases} h = c\Omega + \chi^{1,2} \\ J \wedge \chi^{1,2} = 0 \end{cases} \longrightarrow \mu_{mn} = \mu_{nm}$$

define $\quad \mu_m{}^{\bar n} = \frac{1}{2} \Omega_{mnq} \chi^{\bar n p q}$

$$\underbrace{\partial_{[m} \mu_{n]}{}^{\bar p} - \mu_{[m}{}^{\bar q} \partial_{\bar q} \mu_{n]}{}^{\bar p} = 0}$$

KS equation

μ - deformation of complex structure

and $\quad g^{m\bar n} \nabla_{\bar n} \chi_{m\bar p\bar q} - 4 \chi^n{}_{\bar r\bar s} \chi^{m\bar r\bar s} \nabla_n \chi_{m\bar p\bar s} = 0$

the same as $\quad \nabla^m \mu_{mk} = -\frac{1}{8c} \left(\nabla_k - 8c\, \mu_{km} \nabla^m \right) \log (1 + 16 \det \mu$

Solutions up to third order