Non-abelian DΩ-branes in curved backgrounds: from Matrix diffeomorphisms to a Geometric Myers Effect

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- Introduction
- DΩ-branes and diffeomorphisms
- Matrix diffeomorphisms from Open Strings
- A non-abelian DBI action
- Evidence for a geometric Myers effect
- Outlook & Conclusion
Stringy Geometry

- In String Theory, Geometry is defined by probes.

- "Geometry" is not universal
- Closed vs. Open Strings vs. D-branes
- Open with $B_{NS} \neq 0$
  yields noncommutative geometry
- Open with $F_{RR}^{-1} \neq 0$
  gives rise to the Myers effect
- D-brane geometry, is well-defined.
  i.e. D$\phi$-metric is not renormalized.

Douglas
D-branes in Curved Space

- Question:

What is the geometry of $N$ coincident D-branes?

- Single D-brane effective action: Particle in Curved Space

$$S = \int e^{-\Phi} \sqrt{|G_{ij} \partial x^i \partial x^j + F_{ab}|}$$

- $N$ coincident D-branes in flat space

$$x^i \in \mathbb{R} \sim U(1) \quad \rightarrow \quad X^i \in U(N)$$

$$S = \int \frac{1}{2} \text{Tr} \partial \alpha X^i \partial \alpha X_i + \frac{F^2}{4} - \frac{[X^i X^j]^2}{4}$$

- $N$ coincident D-branes in curved space

$$S = \int \text{tr} G_{ij}(X) \partial x^i \partial x^j$$

\[ \text{to determine this!} \]
Matrix valued diffeomorphisms

- What is the action of N coincident D0-branes in curved space?

One expects

$$S = -m \int \text{tr} \sqrt{g_{ij}(x)} \dot{X}^i \dot{X}^j + \ldots$$

- Ordering? Lore

$$[X^i, X^j] \neq 0$$

- Additional interactions? YES

  (viz. potential)

- Characteristic of curved space:

Invariance under diffeomorphisms

- Recall

  $$S_{\text{part}} = \int \sqrt{g_{ij}(x)} \dot{X}^i \dot{X}^j$$

  $$\delta S_{\text{part}} = 0 \quad ; \quad \delta x^i = \delta^i_j(x) \quad ; \quad \delta g_{ij} = -\delta \epsilon^k \delta_{ij}$$

- Graviton is gauge field of diff.

  Coupling is unique

- Search for a symmetry principle:

  Matrix Diffeomorphisms
Overview

- **What**: the action for D-branes in curved space
- **How**: by requiring diffeomorphism invariance PLUS .... string thy

- **Other approaches**
  - Susy and off-diagonal fluctuations and geodesics  
    - Douglas Kato
    - Ooguri
  - Susy and kappa symmetry  
    - Bergshoeff, de Roo
    - Sevrin
  - (Linear) couplings from Matrix theory  
    - Taylor
    - vRaamsdonk
  - Coset-like approaches from $dN^2$-sigma models  
    - Okawa
    - Ooguri

- **Why**:  
  - Probes of Quantum Geometry (Myers)
  - Connection between D-brane Mechanics and Matrix theory
  - Matrix Models:  
    - $O(N)$ vector models
    - LG models
    - Random Matrix Theory

  Diff invariant version of N-particle QM.
Intermezzo: NC Geometry in String Theory

- STANDARD non-commutative Geometry
  \[ [x^i, x^j] = i \theta^{ij} \]
  \[ \delta [\theta^{ij}] = 0 \]

  Open strings

  \[ x^2 \]

- D-brane non-commutative Geometry
  \[ [x^\mu, x^\nu] = x^{\mu a} x^{\nu b} F_{ab} T^c \]

  Open strings

\[ \star - \text{product?} \]
Contents

- Exact formulation of the problem
  - Matrix DIFFeomorphisms
  - Representations and linear couplings
- An action by way of open strings
  - Characteristics of amplitudes
  - Normal Coordinates and diffeomorphisms
  - Matrix Geometry
  - An action
- Properties
  - Potential / T-duality
  - D-geometry constraints
- Consequences
  - A Geometric Myers effect?
D0-branes coupled to gravity

Open strings

Action exists
Exact formulation

- Given the action

\[ S[g, X] = \int \text{tr} \ G_{ij}(X) \dot{X}^i \dot{X}^j \]
\[ = \int \text{tr} \ (\dot{X}^i \dot{X}^j \dot{X}^k \ldots \dot{X}^n) \frac{\partial^n}{\partial t^n} g_{ij}(0) \]

find the precise orderings/couplings such that

- the action has a single trace \( Tseytin \)
- If \( X^i = \text{diag}(x_1^i, \ldots, x_N^i) \) the reduces to a sum of \( N \) particle actions \( Tseytin \)
- Off-diagonal fluctuations have masses proportional to \( d(x_1, x_N) \) \( Douglas \)
- Classical Moduli space is the symmetric product \( M^N / S_N \) \( Douglas \)
- the action is invariant under \( X \rightarrow X'(X) \) for \( g \rightarrow g' \) \( New \)
- Linear couplings agree with known results (Taylor vR; Oda\( h\)a \( Og\)u\( ri\))
Linearized Coupling and Linearized diffeomorphisms

- Single D-brane (particle)

\[ S = \int d\tau \ g_{ij}(x) \dot{x}^i \dot{x}^j = \int d\tau \ \dot{x}^i \dot{x}^j \ (\eta_{ij} + h_{ij}(x)) \]

\[ \equiv \int d\tau \ \dot{x}^i \dot{x}_i + T^{ij}(x) h_{ij}(x) \]

\[ T_{ij}^{(x)} \frac{\partial^n}{n!} \left. h_{ij} \right|_{x=0} \]

- Current (stress-tensor) \( T^{ij}(x) \) conserved (on shell) "follows" from invariance under infinitesimal diffs:

\[ \delta x^i = \delta^i(x) \quad \delta h_{ij} = -D_i(h) \delta_j - D_j(h) \delta_i \]
Linearized diffs II:

- To first order in $\epsilon$, the action

$$S = \int dt \dot{x}^i \dot{x}_i + T^i_j(x) \delta x^i(x)$$

is also invariant under linear diffs.

\[
\begin{align*}
&\delta x^i = \xi^i(x) \\
&\quad = x^k_0 \ldots x^{un}_0 \frac{\partial^n}{\partial \xi^i} \bigg|_{\xi^i=0}
\end{align*}
\]

\[
\begin{align*}
\delta h_{ij} &= - \partial_i \xi_j - \partial_j \xi_i + O(\epsilon) \\
\delta \partial^n h_{ij} &= - \partial^n \partial_i \xi_j + O(\epsilon)
\end{align*}
\]

- "Reconstruction": to first order the gauge field coupling to a conserved current yields an invariant action iff

$$\delta A_i = - \partial_i \xi$$
Linear coupling to graviton (stress-tensor); known from Matrix theory and string amplitudes (Taylor uR; Ozala Oogur)

\[ S = \int dz \dot{X}^i \dot{X}_i + T_{ij}(k_1, \ldots, k_n) \frac{\mathcal{J}^{(n)}}{n!} h_{ij} + \ldots \]

\[ T_{ij}(k_1, \ldots, k_n) = \text{Str}(\dot{X}^i \dot{X}^j X^{k_1} \ldots X^{k_n}) \]

Different for bosonic string

Stress tensor is conserved (on shell) \[ \Leftrightarrow \]
Invariant under linearized diffeomorphisms

\[ \delta h_{ij} = - \partial_i \delta_j - \partial_j \delta_i + \ldots \]

\[ \delta X^i = \delta^i(X) = \text{Sym} (X^{k_1} \ldots X^{k_n}) \frac{\mathcal{J}^{(n)}}{n!} \]

"Reconstruction" of higher order terms \ldots
Structure of Matrix Diffeomorphisms

*derive by Noether method*

- Very difficult!
- Need to guess 2\textsuperscript{nd} order

\[
\mathcal{S}^{(2)} X = \mathcal{S}^{(2)} = \hbar \mathcal{S} \left[ [X, X], X \right]
\]

- "Normally" need either $\mathcal{S}^{(2)} X$ or $\mathcal{S}^{(1)}$
- Important to note that

\[
\mathcal{S} X = \mathcal{S} \left[ g, X \right]
\]

\[ \uparrow \text{gauge field} \]

- Group DIFF of matrix-valued diffs.

- Lift diff to DIFF
  Group Structure
  \[
  \text{diff} \quad \frac{d}{dX} x''(x'(x)) = x''(x)
  \]
  \[
  \text{DIFF} \quad X''(X'(X)) = U X''(X) U^+ \quad (?)
  \]
- Has NO solution
- $\{ g, X^2 \}$ form a representation
Intermezzo

To implement matrix-valued diffeomorphisms:

- Seek an action that obeys
  \[ S[x, g] = S[x'(g, x), g'(g)] \]

- Extremely non-linear problem

- Brute force (Noether) "fails"

- Can we find some extra information?
D-brane actions from open strings

- D-brane effective action = effective action reconstructed from open string amplitudes.

- Disc amplitudes with $k$ open-string vertices and $n$ graviton vertices

- Diffemorphism invariance of string theory $\Rightarrow$ dift invariance of the effective action

- $n=1$ graviton amplitudes yield linearized result

- Rather than compute amplitudes, try to glean information from the general structure
String Effective Actions

Intermezzo

- Why not use $\beta$-functions?

- $\beta$-functions work for massless fields (marginal operators) in a BG-field method.

- The matrix $X$ corresponds to the open-string field $V$ with vertex

$$V_i(k) \propto Y^i e^{ikY}$$

- Hence we would need "to turn-on" off-diagonal parts of $X$, but these correspond to massive operators.

- Does the LEFA in fact exist?

- Does the matrix structure survive $\alpha' \to 0$?

- Physical expectation YES

- Provided, masses $\langle X \rangle \ll \sqrt{\frac{\alpha'}{4\pi}}$, curvature $R \ll \sqrt{\frac{\alpha'}{4\pi}}$

- Inspection of two-graviton amplitudes suggests a consistent $\alpha' \to 0$ limit exists.
Lessons from open strings

- The action from amplitudes

\[ V_i(h) \, \tilde{\Omega}_n \, Y^i \, e^{i k Y^i} \, ; \, h_{ij}(k) \, dY^i \, dY^j \, e^{i k Y^i} \]

- Action

\[ S = S \left[ h_{ij}(x), \tilde{X}^i \right]_{\eta, \chi} \]

\[ \tilde{X}^i = (\eta + h)^{ij} V^j \]

- Can we relate this to \( S[g, x] \)?

- Vertex \( V_i(k=0) \) \( \Rightarrow \) shift in \( x \)!

- \( V_i \) transform as vectors matrices

\[ \tilde{X}^i \sim A \tilde{X}^i \]

caution: contact terms

- Can we preserve the vector transformation properties?
Normal Coordinates

- **Particle**

- Final action depends on

\[ X = \bar{x} + \tilde{X} + \mathcal{O}(\tilde{x}^2) \]

- \( X \) should transform as a vector,

\[ \Rightarrow \text{ expansion is known: covariant} \]

\[ \text{non-linear BG-field expansion} \]

\[ x = \bar{x} + \bar{x}^i - \sum \Gamma^i_{\alpha \beta} \bar{x}^\alpha \bar{x}^\beta \]

- In that case, all other quantities will be covariant

\[ S[X'(\bar{x}, \bar{x})], X(\bar{x}, \bar{x})] = S [g(\bar{x}) D_{\bar{x}} \cdots D_{\bar{x}}, \bar{x}] \]

- Special Coordinate Choice where non-linear terms are absent

\[ \Gamma^i_{\alpha \beta}(\bar{x}) = 0 \]

\[ \Leftrightarrow \]

\[ X'(t) = \tau \gamma^i \text{ is a solution to the geodesic equation; the field equation of the action.} \]
- Matrix normal coordinates defined by

\[ X^{i \alpha} (t) = t Y^{i \alpha} \]

is a solution to the field equation.

- This fixes regular plus "new" diffs

\[ S_X \sim [X, X] \]

- Action will contain \( dN^2 \)-dim covariant quantities

\[ S = S \left[ G_{ij}, D_k, \ldots, D_{kn} R, \vec{x}^I \right] \]

which are functionals of \( d \)-dim covariant quantities \( g_{ij} \) & \( D_k, \ldots, D_{kn} R \) !!
Normal Coordinates & Diffeomorphisms

• Every action appears DIFF invariant?! as the action, for any ordering
  \[ S \left[ g_{ij}(x), Dk_i ... Dk_n R(x), \tilde{X} \right] \]
  only depends on covariant quantities

• Normal coordinates are based on a special point \( x \); difft invariance means independence of the base point \( x \)

Transformation to new normal coordinates must leave action \( S \) invariant !!
  - difft translates in the symmetry
    \[ SS = 0 \quad \text{"} Sx = \varepsilon \]
    \[ SD_i ... DR = \varepsilon Di D_i D_i ... DR \]
    \[ S \tilde{X} = \varepsilon - \Gamma^{\alpha}(\tilde{X}) \varepsilon \]
  \( \uparrow \) solution to geodesic equation = field equation of \( S \)!!

- imposes that the action only depends on \( X = \varepsilon + \tilde{X} \)
Shift Symmetry in BG field formalism

- BG-field formalism in YM theory

- BG-Quantum Split

\[ S(A_\mu + \phi_\mu) = D_\mu (A + \phi) \Lambda \]

\[ \Rightarrow \quad S A_\mu = D_\mu (A) \Lambda \\
S \phi_\mu = [\phi_\mu, \Lambda] \]

- Shift Identity/Symmetry

\[ S A_\mu = \varepsilon A_\mu \quad \text{and} \quad S \phi_\mu = -\varepsilon \phi_\mu \]

(Covariant) BG-field formalism in NLO YM

- **NONLINEAR** BG-Quantum Split

\[ S (x + \xi - \ldots) = \varepsilon (x + \xi - \ldots) \]

\[ \Rightarrow \quad S x^\mu = \varepsilon x^\mu(x) \\
S \xi^\mu = -\varepsilon \nabla^\mu \xi^\nu \varepsilon^\nu \]

- **NONLINEAR** Shift symmetry (combined with change to new RNC)

\[ S (x + \xi) = x + \xi - \Gamma^{(m)} \chi \]

\[ \Rightarrow \quad S \xi = \varepsilon + \chi + \varepsilon \nabla \chi \]
An Algorithm for Diffs

- Diffs are maps Normal Coord$_1$ $\rightarrow$ Normal Coord$_2$ that have advantage, that invariance can be imposed algorithmically.

- Write action $S$ with all possible terms.

- Impose Normal Coord$_1$ by requiring
  \[ X^R = \varepsilon Y \]

  is a solution to field eq $\Rightarrow$ constraints$_1$.

- Change to Normal Coord$_2$ around $\varepsilon$ by solving field eq order by order; substitute in action $S$ and require that

  \[ S_{NC2} [g, D, DR, \vec{X}, \varepsilon] = S_{NC1} [g, e\partial, D, R, \vec{x}] \]

  $\Rightarrow$ constraints$_2$.

- Solve system of constraints $1$ and $2$. 
Normal coordinates & Matrix Geometry

- Expand the action \((dN^2 - \text{sigma model})\)

\[
S = \mathcal{G}_{IJK} \tilde{X}^I \tilde{X}^J = \mathcal{G}^{i\alpha j\beta k\gamma} \tilde{X}^i \tilde{X}^j \tilde{X}^k \tilde{X}^\gamma
\]

in normal coordinates

- Numerical curvature tensors \(R_{IJKL}(g, R_{ijkl})\) are functionals of \(d - \text{curvature tensors}\).

- These functionals must obey the usual props.

- Impose on \(R_{IJKL}, \nabla_I R_{JKLM}, \text{etc}\)

  a) \(\tilde{R}_{IJKL} = - R_{IJKL} = R_{JKIL}\)

  b) \(\tilde{R} = \tilde{R}_{IJKL} = 0\)

  c) \(\nabla_I R_{JKLM} = 0\)

  d) \([\nabla_I, \nabla_J] R_{KLMN} = R_{IJK}^{\phantom{IJK}T} R_{LMN} + \ldots\)

  - etc.

  e) \(U(N)\) indices contracted in a single trace

  f) Basepath invariance

  \[
  \mathcal{G}^\alpha_{\beta \phi} \nabla_i \mathcal{G}^\alpha_{\phi \beta} = \nabla_i
  \]

  g) To first order, symmetrized result

  h) Correct \(U(1)\) limit
Solution

- Constraint e) implies that

\[ R_{ijkl} = R_{ijkl}' \Delta x' y' + \text{orderings}, \]
\[ \Delta x' y' = \delta_{x'y'} \delta_{x'y}, \delta_{x'y}, \delta_{x'y}, \delta_{x'y} \]

- At order 4, \( R_{ijkl} \), and 5, \( \nabla_i \nabla_j R_{klmn} \)
  symmetrized result. Order 6 requires new
  structures due to \( [\nabla_i, \nabla_j] R_{klmn} \) condition.

- Answer: 120 possible terms

  \[ \downarrow \]
  \[ \text{32-dim space of solutions} \]

\[ \text{Tr} \left( \chi^n X^m X^i X^j X^k X^l \right) T_{\mu \nu \mu \nu i \nu j \nu k \nu l}^{\text{4}} \]
\[ + \text{Tr} \left( \chi^n X^m X^i \dot{X}^j X^k \dot{X}^l \right) T_{\mu \nu \mu \nu i \nu j \nu k \nu l}^{\text{2}} \]
\[ + \text{Tr} \left( \chi^n X^m X^i \dot{X}^j \dot{X}^k X^l \right) T_{\mu \nu \mu \nu i \nu j \nu k \nu l}^{\text{3}} \]

\[ T_{\mu \nu \mu \nu i \nu j \nu k \nu l}^{\text{4}} = \left( -\frac{7}{120}, -2\beta \right) R_{\mu i k l} R_{\nu j} \gamma^{\nu} + 23 \text{ terms} \]

\[ T_{\mu \nu \mu \nu i \nu j \nu k \nu l}^{\text{2}} = \frac{1}{120} \text{ R}_{\mu j k l} R_{\nu i} + 31 \text{ terms} \]

\[ T_{\mu \nu \mu \nu i \nu j \nu k \nu l}^{\text{3}} = \left( -\frac{47}{120} - \frac{\alpha}{5} - \frac{6\beta}{5} \right) R_{\mu i k l} R_{\nu j} \gamma^{\nu} + 15 \text{ terms} \]

- No obvious structure emerges
Properties of the Solution

- There are new vertices cp. with UK1:
  - $U(1)$ only $(R \bar{x})(R \bar{x})$
  - $U(N)$ e.g. from $T^2$: $R_{mjk} X^{j} X^{k}$
  - Appears to hold for all values of the 32 parameters.

- The construction of the solution obscures that some of the 32 parameters will be fixed at higher order
  - e.g. $\Delta S = Tr(Asym(\dot{X}^{j} X^{k} X^{l}) Asym(\dot{X}^{m} X^{n} X^{p}))$
    $R_{ij} X^{l} R_{jkl}$
    is left unfixed at this order.
  - Unlikely all of them will be fixed at higher order
    - Bosonic string & superstring are different reps of Matrix DIFF, expect more.
The potential, fermions, other contributions

- Need potential for remaining two constraints of D-geometry.

- Introduce vielbeins

\[ g^{ij} = E^A_i E^B_j \quad E^A_i = E^{ix} \]

- $U(N)$-beams

\[ g^{ij} \delta_{jy} = E^{ix}(x) E^{iy}(x) \]

\[ \text{compatible with single trace} \]

- $\text{Diff} \rightarrow SD(DN^2)$ transformation of $E^{ix} \dot{X}^i$

i.e. $\text{Tr} E^A_i \dot{X}^i$ transforms nicely

- Note that $[X, ...]$ acts as $d/ds$

implies that $\text{Tr} (E^A_i \dot{X}^i V)$ transforms nicely

- Some guesswork leads to the potential

\[ V = \frac{1}{2} \text{Tr} [X^i, X^j]^2 \rightarrow \]

\[ = \frac{1}{4} \text{Tr} \left[ E^A_i, X^i \right] \left[ E^B_j, X^j \right] \left[ E^C_k, X^k \right] \left[ E^D_l, X^l \right] \]

- Also yields correct linearized answer
Spectrum of Quadratic Fluctuations

- Remaining constraints of D-Geometry
  - Quadratic fluctuations $\lambda$ have masses $m_\lambda \sim d(x_i, x_j)$
  - Classical Moduli space is $M^N/S_N$

- Quadratic fluctuations
  - Diff invariance + normal coordinates imply that we only need to check
    $V|_{x^i = 0} \sim d(0, x) = g_{ij}(x) A^i A^j$
    - Checks out!!

- Classical Moduli space $M^N/S_N$ also follows.
Fermions and Susy

- Need vielbeins for susy/fermions
  - Appearance of vielbeins in potential is encouraging

- Susy constraints ?!
  - Susy on the worldvolume should follow from kappa symmetry
  - For kappa symmetry
    \[ SS^{\text{kin}} = - SS^{WZ} \]
    need \( WZ \) - term.
  - Do additional constraints arise?
    For non-abelian kappa-symmetry
    only closes on-shell ↑
Gravitational Myers Effect

The Myers effect

- In the presence of RR 4-form flux

\[ V(X) = \frac{1}{4} \text{Tr} \left[ X^i X^i \right]^2 - F_{ijkl} \text{Tr} X^i X^j X^k \]

- Two solutions

\[ \frac{\partial V}{\partial X} = 0 \]

\[
\begin{cases}
X^i = \text{diag} x_i^i \\
[X^i, X^k] = i F_{ijkl} X^j
\end{cases}
\]

- The non-trivial solution corresponds to a polarized D2-brane.

Signal for Myers effect is the presence of a tachyonic mode in the spectrum of quadratic fluctuations around the trivial solution.

Jatkar et. al.
Gravitational Myers Effect II

- Question: Is there a Gravitational Myers effect?

- Is there a non-trivial critical point for the action

\[ S(X) = \frac{1}{2} \text{Tr} \, E^A_i \dot{X}^i \text{Tr} \, E_j^A \dot{X}^j - \frac{1}{4} \text{Tr} \, [E, X]^4 \]

- Highly nonlinear
- Difficult to find solutions

- Signal of Gravitational Myers effect: spectrum of quadratic fluctuations

- CONFLICT?
  - D-Geometry: off-diagonal fluctuations have \( m^2 = d^2(x, y) > 0 \)
Gravitational Myers effect

- "Trivial" Solution is $N$ freely falling non-interacting particles.

\[ X^b = \text{diag} \left( x^i_x \right) \]

with

\[ \ddot{x}^i_x + \nabla^i_jk \dot{x}^i_x \dot{x}^j_x \dot{x}^k_x = 0 \]

for each entry $x^i_x$.

- In D-Geometry, ultra-trivial background

\[ \dot{x}^i_x = 0 \]

- Myers effect possible iff $\dot{x}^i_x \neq 0$

  Connection w. Giant Graviton?

- Quadratic fluctuations?!

- Single Particle:

\[ S S = S x^i M_{ij} S x^j \]

\[ M_{ij} = R_{i m j} \dot{x}^m \dot{x}^n + \nabla_v \nabla_v \]

- $M_{ij}$ is the kernel of the geodesic deviation operator; measures tidal forces!
Gravitational Myers effect

- Spectrum of Quadratic off-diagonal fluctuations
  (around N-freely falling particles)

\[ S^{(2)} S = S^{(2)} S^{\text{kin}} + S^{(2)} S^{\text{pot}} \]

\[ \implies S^{(2)} S^{\text{pot}} = -\frac{1}{4} \left[ x^i, Y^j \right] \left[ x^i, Y^j \right] \]

\[ S^{(2)} S^{\text{kin}} = \text{Tr} \ Y^i H_{ij} Y^j \]

\[ H_{ij} = R_{mijn} x^m x^n + \nabla_x \nabla_x g_{ij} \]

- Myers effect occurs when tidal forces < kinetic term
  balance internal forces < potential term

Does this happen? \[ \square \text{Yes!} \]

When ambient space is negatively curved locally

\[ g_{ij} x^i x^j \ll \frac{\xi^2 \varepsilon^2}{L^2} \]
Non-commutative Geometry

- Standard non-comm geometry
  - Open strings with $B_{NS} 
eq 0$
    \[ f \cdot g \rightarrow f \star g = e^{iB_{NS} \cdot x} \int f(x)g(y) \]

- Close relations
  - Amplitude calculations both exhibit "Wilson lines"; linear action can be written as
    \[ S = \oint d\lambda \, b_{ij}(\lambda) \, Tr(e^{i \lambda X^i X^j} \lambda X^e \lambda X^j) \]
  - Deformation quantization; diff's preserving $B_{NS}$
    \[ \Rightarrow \text{non-comm gauge transformations} \]
  - 1-1 Map $F = B + F \leftrightarrow [X, X]$
Outlook and Conclusions

- Have succeeded in finding a matrix-diffeomorphism invariant action
  - Constructed "Matrix-Geometry"
  - Presented a solution to order 6

- Have glimpsed at the intricate geometrical structure
  - ... obvious patterns still lacking
  - Connection with non-commutative geometry

- Open questions and further directions
  - WZ term; kappa & supersymmetry
  - Explicit Computational Myers effect
  - Applications; Schwarzschild
  - Other closed String backgrounds
  - Fundamental Symmetry structure
    - Matrix theory?
  - Non-Commutative YM? v Raamsdonk