



# One in a Billion: MSSM-like D-Brane Statistics

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  - Motivation
  - Type II orientifold models
- 2 Methods of statistical analysis
  - Computer search
  - Number of solutions
- 3 Results
  - Full set of models
  - Searching for MSSM-like models
  - Results for Pati-Salam models
  - Statistics of the hidden sector
  - Gauge couplings
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- 4 Conclusions



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- **Statistical approach** to string vacuum problem

[Ashok, Denef, Douglas, Shiffman, Zelditch; De Wolfe, Giryavets, Kachru, Taylor, Tripathi; Misra, Nanda; Conlon, Quevedo; Kumar, Wells; Dine, Gorbatov, Thomas, O'Neil, Sun; Dienes, Dudas, Gherghetta; Acharya, Denef, Valandro]

- Analysis of the **gauge sector** in a specific setup
- **Distribution** of SM-like properties in these models
- **Correlations** between observables



- Type IIB orientifold flux compactifications
- Analysis of the ( $\mathcal{N} = 1$ ) gauge sector
- RR/NSNS 3-form fluxes to freeze complex structure moduli and dilaton
- Add magnetized D-branes to cancel tadpoles and get chiral fermions
- In the special class of orbifolds we are considering the consistency conditions are well under control



## Type IIA/ $\Omega\bar{\sigma}$ with D6-branes at angles

- D-branes wrap sLag 3-cycles
- Symplectic basis:  $(\alpha_I, \beta_I)$  of  $H_3(M, \mathbb{Z})$ , where  $\alpha_I \in H_3^+(M)$  and  $\beta_I \in H_3^-(M)$

- **O6-planes:**

$$\pi_{O6} = \frac{1}{2} \sum_{I=1}^{b_3/2} L_I \alpha_I$$

- **D6-branes:**

$$\pi_a = \sum_{I=1}^{b_3/2} (X_{a,I} \alpha_I + Y_{a,I} \beta_I),$$

$$\pi'_a = \sum_{I=1}^{b_3/2} (X_{a,I} \alpha_I - Y_{a,I} \beta_I)$$



## Tadpole cancellation

$b_3/2 = 1 + h_{21}$  conditions:

$$\sum_{a=1}^k N_a X_{a,I} = L_I - L_{I,flux}$$

## Supersymmetry conditions

- sLag condition:  $\Im(\Omega_3)|_{\pi_a} = \sum_{I=1}^{b_3/2} Y_{a,I} F_I(U) = 0$ , where  
 $F_I = \int_{\beta_I} \Omega_3$

- anti-branes:  $\Re(\Omega_3)|_{\pi_a} = \sum_{I=1}^{b_3/2} X_{a,I} U_I > 0$



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## K-theory constraints

[Uranga; Marchesano, Shiu]

$$\sum_a N_a Y_{0,a} \in 2\mathbb{Z}$$

- Number of solutions changes by a factor of 6
- Models which have an odd rank of the gauge group are suppressed  
 $\rightsquigarrow$  rank-distribution changes. ( $rk = \sum_a N_a$ )



We get chiral matter at the **intersection** of D-branes

$$\begin{aligned} I_{ab} &= \pi_a \circ \pi_b \\ &= \sum_I X_{a,I} Y_{b,I} - Y_{a,I} X_{b,I} \end{aligned}$$

$\rightsquigarrow I_{ab}$  chiral multiplets in a bifundamental  $U(N_a) \times U(N_b)$  representation.



Orbifold:  $T^2 \times T^2 \times T^2/\mathbb{Z}_2 \times \mathbb{Z}_2$

[Cvetic, Shiu, Uranga]

- $(h_{1,1}, h_{2,1}) = (51, 3)$
- Wrapping numbers  $(n_I, m_I), I \in \{1, 2, 3\}$
- Tilted tori:  $m_I \rightarrow m_I + b_I n_I, \quad b_I \in \{1/2, 1\}$
- Define

$$X_0 = n_1 n_2 n_3, \quad X_1 = -n_1 m_2 m_3, \quad X_2 = -m_1 n_2 m_3, \quad X_3 = -m_1 m_2 n_3, \\ Y_0 = m_1 m_2 m_3, \quad Y_1 = -m_1 n_2 n_3, \quad Y_2 = -n_1 m_2 n_3, \quad Y_3 = -n_1 n_2 m_3,$$

satisfying

$$X_I Y_I = X_J Y_J \quad \forall I, J, \quad X_I X_J = -Y_K Y_L, \\ X_L (Y_L)^2 = -X_I X_J X_K, \quad Y_L (X_L)^2 = -Y_I Y_J Y_K \quad I, J, K \text{ cyclic}$$



# Constraints from consistency conditions

- SUSY:

$$\sum_{I=0}^3 Y_I U_I^{-1} = 0, \quad \text{ans} \quad \sum_{I=0}^3 X_I U_I > 0,$$

- Tadpole cancellation:

$$\sum_a N_a X_{a,I} = L_I, \quad I \in \{0..4\},$$

- Combined:

$$0 < \sum_{I=0}^3 X_I U_I \leq \sum_{I=0}^3 L_I U_I$$

In our models:  $L_0 = 8 - N_{flux}$ ;  $L_i = 8$ ,  $i \in \{1, 2, 3\}$ .  
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# Constraints from consistency conditions

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## Problem

Find solutions of diophantine equations of the form

$$\sum_{a=1}^k N_a X_a^I = L^I$$

## Solution

- 1 Choose values for  $U_I$ . Generate sets of  $X^I$  that fulfill SUSY conditions.
- 2 Count number of partitions  $\sum_{a=1}^k S_a = L^I U_I$ .
- 3 Factorize  $S_a = N_a (X_a^I U_I)$  using values for the  $X_a$  from step 1.
- 4 Check if tadpole and K-theory conditions are fulfilled.



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Performed using a fast partition/factorization algorithm for natural numbers.

- With the help of a computer cluster ( $> 500$  processors running for  $\approx 6$  months) all possible solutions (for a sufficient range of  $U_I$ ) have been generated

$\rightsquigarrow$  almost complete classification of models on  $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ .

- In total  $\sim 10^8$  models have been analysed



# Counting solutions

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- Susy and tadpole conditions allow only for three sets of  $X_I$ : Only stacks with 1, 2, or 4 non-vanishing  $X_I$  are possible.
- In the first two cases one can prove that the number of solutions is finite.
- In the last case we get  $X_A = - \left( \sum_i \frac{U_A}{U_i X_i} \right)^{-1}$ .  
↪ for a sufficient number of these branes the complex structures are fixed at rational values

$$1 \leq X_i \leq \sum_{P=0}^3 \frac{u_{P,2} u_{Q,1} u_{R,1} u_{S,1} L_P}{u_{i,2} u_{J,1} u_{K,1} u_{L,1}}$$

↪ for fixed complex structures only a finite number of branes are admissible

- **Computer analysis:** number of solutions decreases rapidly for high values of the complex structures



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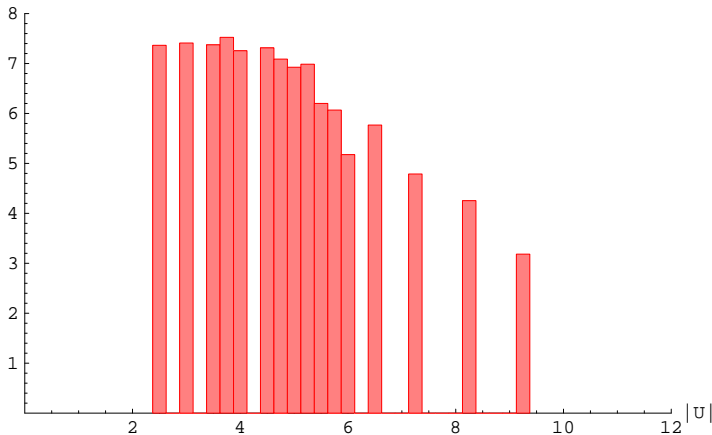
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Log(# models)



Number of models computed depending on the absolute value of our complex structure variables.





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# Total rank distribution

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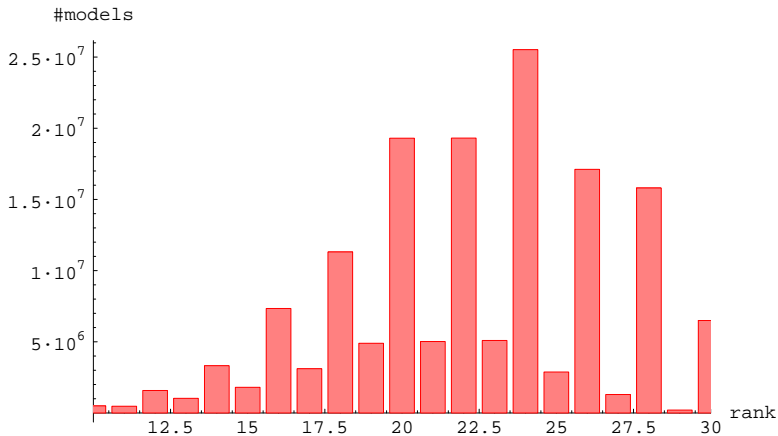
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# Distribution of $U(M)$ gauge groups

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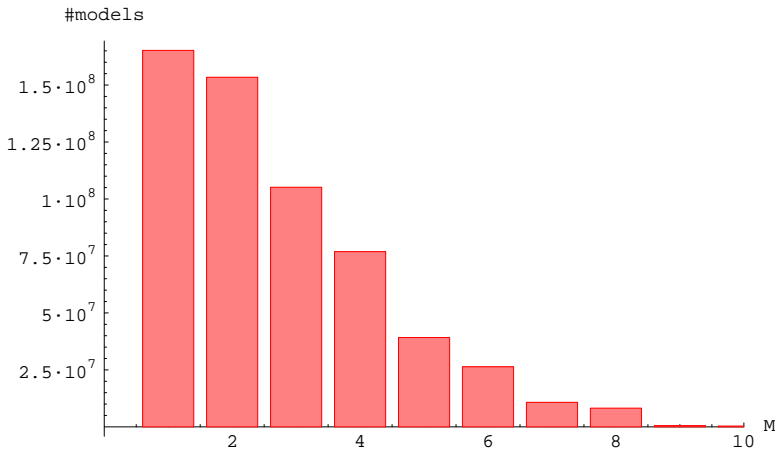
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- As a measure for overall **chirality** we consider

$$\chi = \sum_{a>b} I_{a',b} - I_{a,b} = 2 \vec{Y}_a \vec{X}_b$$

- **Odd** values for  $\chi$  are possible only from tilted tori.



# Chirality distribution

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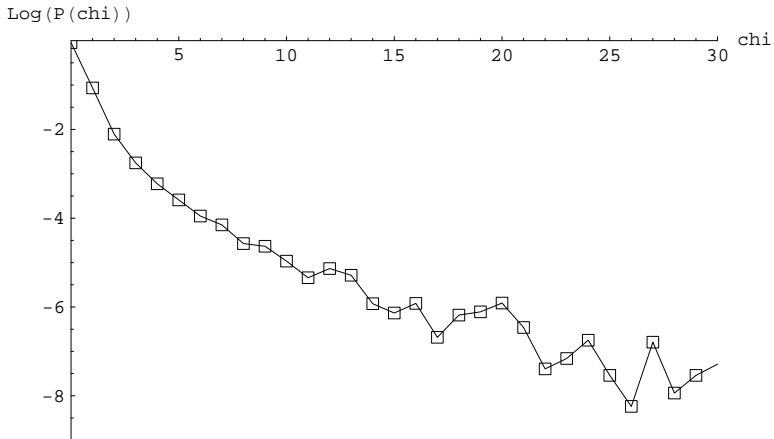
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Logarithmic plot of frequencies of given mean chirality.



# Rank vs chirality

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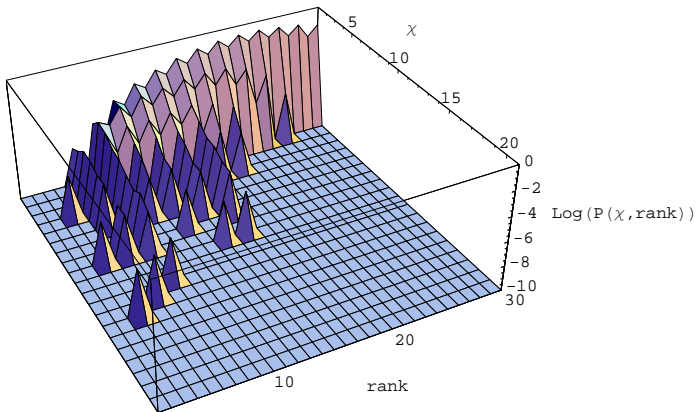
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Distribution of models with fixed rank and chirality.



**Aim:** Systematic investigation of distribution of vacua with standard model-like characteristics among set of SUSY solutions.  $\rightsquigarrow$  MSSM realized on 4 or 3 stacks of branes

- **4 stacks:**  $U(3)_a \times U(2)_b / Sp(2)_b \times U(1)_c \times U(1)_d$

**QCD**  $U(3)_a = SU(3)_{QCD} \times U(1)_a$

**weak**  $U(2)_b = SU(2)_w \times U(1)_b$

$U(1)_Y$ : appropriate (massless) combination  $Q_Y = \sum x_i Q_i$

- **3 stacks:** possible if  $x_c = x_d$  by dropping stack  $d$  in 4 stack solution



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We want a **chiral spectrum** on intersections of MSSM branes

$$\#(N_a, \overline{N}_b) = \pi_a \circ \pi_b$$

$$\#\text{Anti}_a = \frac{1}{2}(\pi_a \circ \pi_{a'} + \pi_a \circ \pi_{O6})$$

$$\#\text{Sym}_a = \frac{1}{2}(\pi_a \circ \pi_{a'} - \pi_a \circ \pi_{O6})$$

$\rightsquigarrow$  systematic realization of MSSM quantum numbers



# Anomaly considerations

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- Freedom of non-abelian anomalies guaranteed by RR-tadpole cancellation
- $U(1)_a - SU(N_b)^2$  mixed anomalies cancelled by GS mechanism, but have to make sure that specific realization of  $U(1)_Y = \sum x_i U(1)_i$  is anomaly free and does not receive mass by GS-coupling, i.e.

$$\sum_a x_a N_a Y_I^a = 0, \quad I = 0, \dots, 3$$



# Number of generations

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- **No** MSSM configurations with **three** generations and massless  $U(1)$  have been found in the analysed data.
- **Reason:** All known solutions work with values for the complex structure which is outside the range we considered. [Cvetic et al.; Marchesano, Shiu]
- $\rightsquigarrow$  These models are statistically **highly suppressed**.



# Distribution of generations

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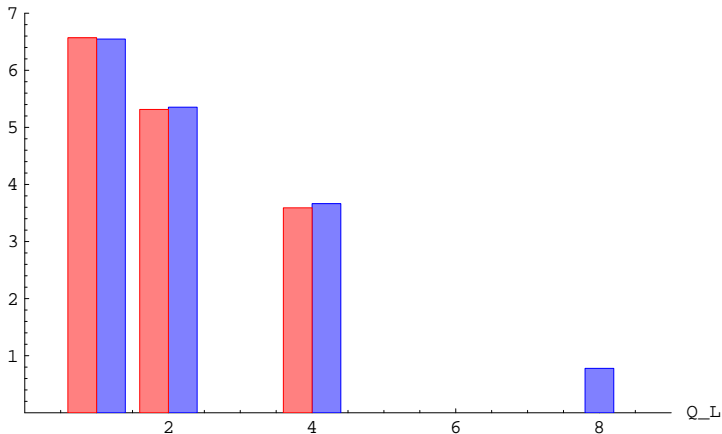
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Log(# models)



Number of models found for MSSM configurations (red), allowing for a massive  $U(1)$  (blue).



In addition we consider possible Pati-Salam models with gauge group

$$SU(4) \times SU(2)_L \times SU(2)_R$$

[see e.g. Cvetic et al.]

- **Condition:** Intersection numbers between  $SU(4)$  and both  $SU(2)$  stacks have to be equal
- **Number of generations:** We did **not** see **three** generation models (for the same reason as in the MSSM case)



# Distribution of generations

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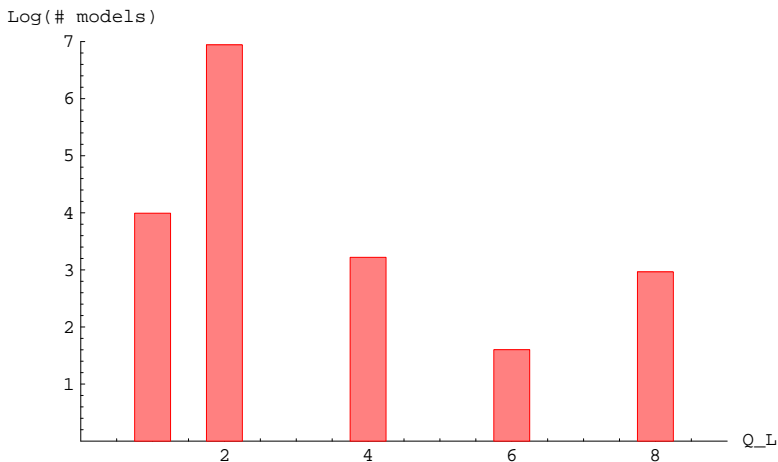
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Number of models found for Pati-Salam configurations.



# Hidden sector

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Besides the standard model or Pati-Salam gauge group we have in general a hidden sector

$$G = G_{SM} \oplus H.$$

## Question

Are the distributions in the hidden sector different from the distributions in the full set of models?

## Answer

**No**, they are not. The distribution of gauge group factors or chirality in the hidden sector of standard model or Pati-Salam configurations is basically the same as in the full set of models.

↪ Some properties of our models might be **generic** in the sense that they do not depend on the constraints for the visible sector.



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# Rank distribution in the hidden sector

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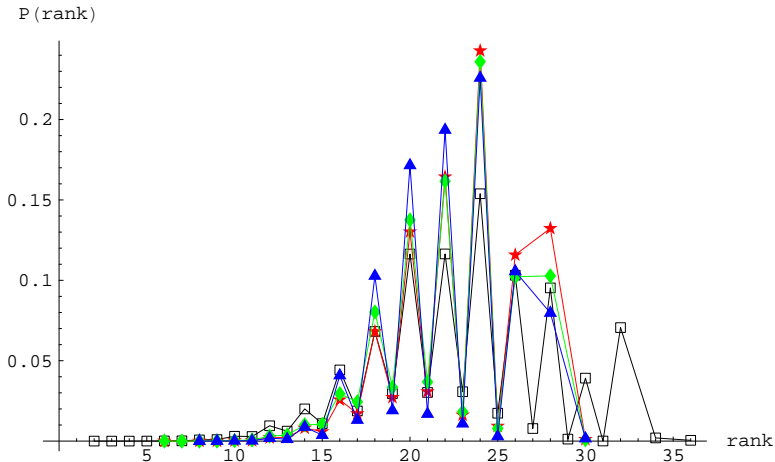
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Black boxes: All models, red stars: MSSM (massless  $U(1)$ ), green diamonds: MSSM (all  $U(1)$ ), blue triangles: Pati-Salam.



# $U(M)$ distribution in the hidden sector

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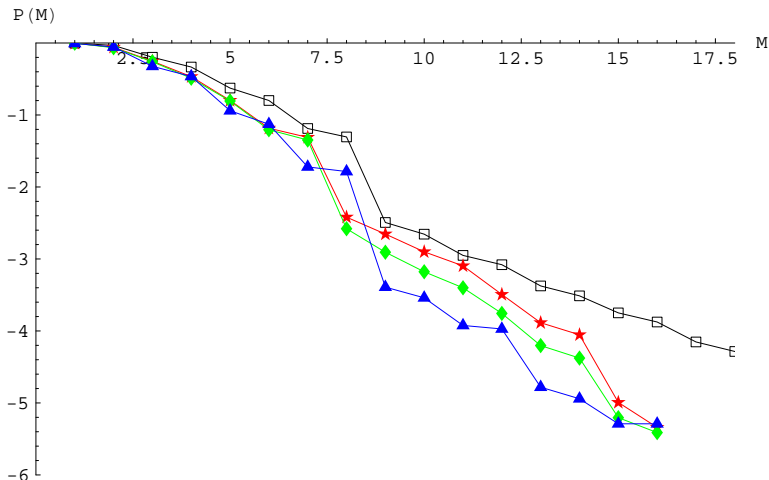
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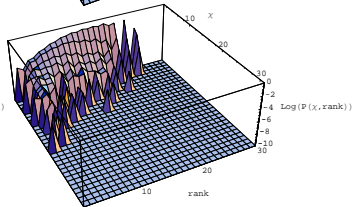
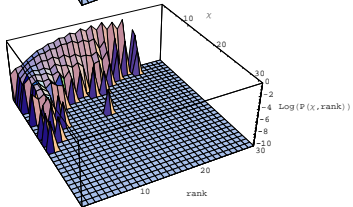
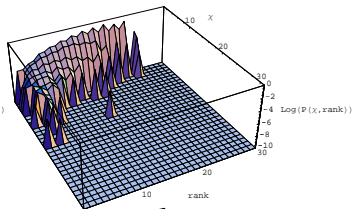
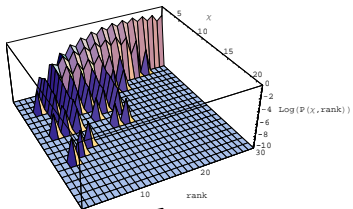
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Form top-left to bottom-right: All models, MSSM-like, massive MSSM, Pati-Salam.



# Gauge couplings

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So far we considered properties of the models which are topological, in the sense that they depend on the brane configuration only. A geometric quantity one might consider are the **gauge couplings**  $\alpha_s$ ,  $\alpha_w$  and  $\alpha_Y$ .

In principle they should be considered at **low energy**, but this would imply that we use the renormalization group to evolve them down from their values at the **string scale**.

↪ We do not do this, but consider their string-scale values just to get some hints about their behaviour.



## Calculation of the gauge couplings

$$\frac{1}{\alpha_a} = \frac{c}{\kappa_a} \frac{1}{\hat{b} \sqrt{\prod_{i=1}^3 R_1^{(i)} R_2^{(i)}}} \sum_{I=0}^3 \hat{X}^I U_I,$$

with some normalization constant  $c = \frac{1}{2\sqrt{2}} \frac{M_{Planck}}{M_s}$  and  $\kappa_a \in \{1, 2\}$ , depending if we have a  $U(N)$  or  $SO(N)$  stack.

**Note:** Explicit dependence on the complex structure.

## Conjecture about relations

There exists a conjecture that (most) intersecting brane models should obey the relation [Blumenhagen, Stieberger, Lüst]

$$\frac{1}{\alpha_Y} = \frac{2}{3} \frac{1}{\alpha_s} + \frac{1}{\alpha_w}.$$



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# Distribution of $\alpha_s/\alpha_w$

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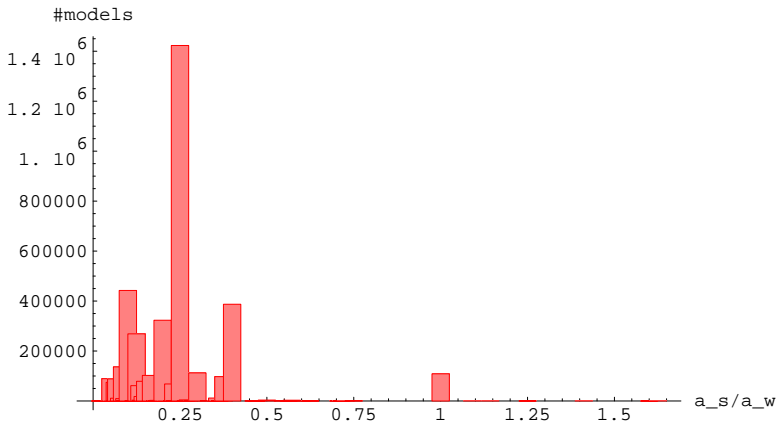
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The Weinberg angle  $\sin^2\theta$  is given by

$$\sin^2\theta = \frac{\alpha_Y}{\alpha_w + \alpha_Y}.$$

If the conjectured relation between the coupling constants is correct we would have the following relation between  $\sin^2\theta$  and  $\alpha_s/\alpha_w$

$$\sin^2\theta = \frac{3}{2} \frac{1}{\alpha_w/\alpha_s + 3}.$$

Result

88% of all models fulfill this relation.



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# Values for $\sin^2 \theta$

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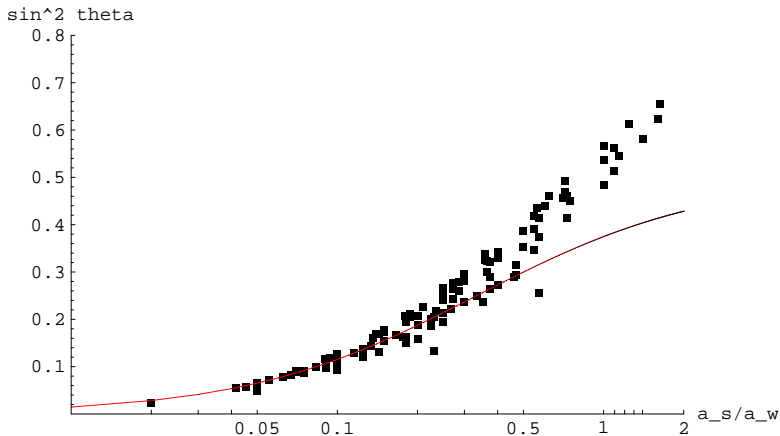
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Logarithmic plot of  $\alpha_s/\alpha_w$  against  $\sin^2 \theta$ . Each dot represents a class of models with the same values.

The conjectured relation is shown as a red curve.



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## Question

Are the atomic properties of the models (like existence of certain gauge groups, total rank, chirality, etc.) **correlated**?

## Answer

**Yes**, some of them are, **but only global** properties, for example the mean chirality and the total rank of the gauge group.



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## Conjecture

Correlations between different constraining properties are **very small**. If this is true, it would be possible to make predictions about the probability to find models with specific properties without constructing them.

How to check this?

Calculate correlation between different properties

$$\frac{|P(1 \wedge 2) - P(1)P(2)|}{P(1 \wedge 2) + P(1)P(2)}$$



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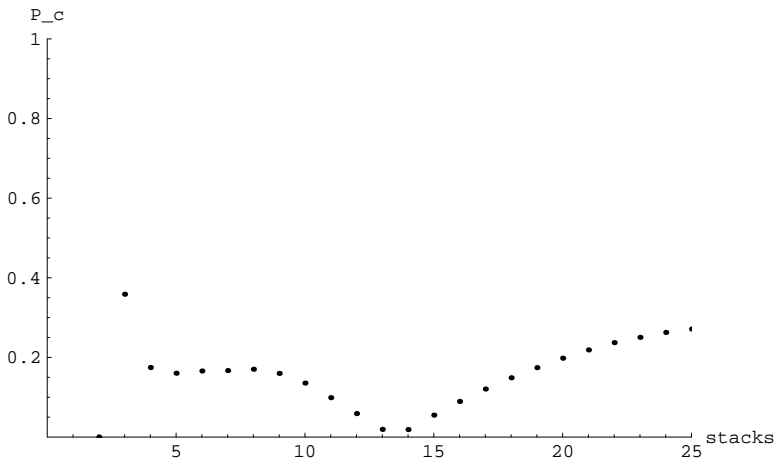
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Correlations between the existence of  $U(3)$  and  $U(2)/Sp(2)$  gauge groups.





# Estimate of three generation MSSMs

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Using this we can try to estimate the number of standard models with three generations in the complete setup, although we have not explicitly constructed a single one.

Restriction	Factor
gauge factor $U(3)$	0.0816
gauge factor $U(2)/Sp(2)$	0.992
No symmetric representations	0.839
Massless $U(1)_Y$	0.423
Three generations of quarks	$2.92 \times 10^{-5}$
Three generations of leptons	$1.62 \times 10^{-3}$
<b>Total</b>	$1.3 \times 10^{-9}$

(matches with Gepner model constructions from [Schellekens et.al.](#))



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## Question

### How good is this estimate?

Compare with estimates done in the same way for models with 2 or 4 generations, where we have exact results.

# generations	# of models found	estimated #
2	162921	188908
3	0	0.2
4	3898	3310

## Answer

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  - Type II orientifold models
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  - Computer search
  - Number of solutions
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  - Correlations
- 4 Conclusions



## Summary

- Standard model configurations are heavily suppressed from a statistical point of view.
- There exist non-trivial overall statistical correlations between physical observables.
- The constraining properties of realistic models are quite uncorrelated.  
↪ features of distributions of sm-properties can be estimated without explicit calculation.

## Outlook

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Thank you!