The Kerr/CFT Correspondence
Holography for Real World Black Holes

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Holography and Black Holes

Motivation 1: BH Information Paradox

- 1960s: Black Hole “Mechanics”

- 1970s: Hawking radiation / Thermodynamics
  - BH evolution is non-unitary in effective field theory.

- 1990s: AdS/CFT
  - Dual CFT description is clearly unitary

Conclude: The low energy description of string theory / quantum gravity is not what it seems, i.e. local effective QFT + general relativity.
Motivation 2: Observations of Black Holes

- Holography has led to a better understanding of black holes in string theory (SUSY, extra dimensions, etc.)

- But what can we learn about real-world black holes observed in the sky?
Kerr Black Holes

- 4d rotating black hole
- Extremal limit: $J = M^2$

- GRS 1915+105: $J \sim 0.99M^2$
  
  McClintock et al. 2006

- Bekenstein-Hawking Entropy
  
  $$S_{\text{ext}} = \frac{\text{Area}}{4} = 2\pi J$$
Main Result

Near the horizon of an extremal Kerr black hole, any consistent theory of quantum gravity is dual to a 2D conformal field theory.

Central charge:  \( c = 12 \, J \)

Derivation: states transform under a Virasoro algebra (i.e., in representations of the 2D conformal group).

Applies to astrophysical black holes (and more).

Things we don’t need:
- Charge
- Anti de Sitter space (AdS)
- Extra dimensions
- Supersymmetry
- String theory
Outline

• Overview
• Asymptotic Symmetries
• Entropy
• Generalizations and applications
  • Charge
  • Anti de Sitter space (AdS)
  • Extra dimensions
  • Supersymmetry
  • String theory
AdS$_3$/CFT$_2$

- Explains every entropy calculation in string theory, eg entropy of 5d black holes. (Strominger, Vafa '95)
- But, complexities of string theory are not needed. (Strominger '97)
- Brown & Henneaux ('86) showed quantum gravity on AdS$_3$ is dual to a CFT with central charge.

\[ C = \frac{3\ell}{2G} \]

- Method: Asymptotic Symmetry Group (ASG)
Near horizon limit:
\[ ds^2 = 2J\Omega^2 \left( -r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2 (d\phi + r dt)^2 \right) \]

AdS$_2$

\[ \Omega^2, \Lambda^2 = \text{functions of } \theta \]
\[ \phi \sim \phi + 2\pi \]

Isometries:
\[ U(1)_L \text{ rotating } \phi \]
\[ SL(2, R)_R \text{ acting on the AdS}_2 \]

Bardeen, Horowitz '99
Asymptotic Symmetries I

★ Asymptotic Symmetry Group [example: $U(1)$ gauge theory]

\[
\text{ASG} = \frac{\text{Allowed symmetries}}{\text{Trivial symmetries}}
\]

★ “Allowed” = obeying the boundary conditions

★ “Trivial” = corresponding charge vanishes
Asymptotic Symmetries II

★ Find allowed diffeos:

\[
\begin{align*}
\zeta_t &= \partial_t \\
\zeta &= \epsilon(\phi)\partial_\phi - r\epsilon(\phi)\partial_r
\end{align*}
\]

★ Generators \( \zeta_n \) with \( \epsilon_n = e^{in\phi} \) satisfy a Virasoro algebra,

\[
i\{\zeta_m, \zeta_n\}_{L.B.} = (m - n)\zeta_{m+n}
\]

★ Associated charges \( Q_n(g_{\mu\nu}) \) are boundary integrals

\[
Q(\zeta, g) = \int_{\partial\Sigma} k[\zeta, g]
\]

★ Supplemental boundary condition \( M^2 = J \) (extremality)
Central charge

★ Compute Dirac brackets

\[ \{ Q_m, Q_n \}_D.B. = \delta_n Q_m \]

★ Result is the Virasoro algebra,

\[ i\{ Q_m, Q_n \}_D.B. = (m - n)Q_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n} \]

⇒ quantum gravity on NHEK is holographically dual to a 2d CFT with

\[ c = 12 \text{ J} \]

GRS 1915+105 \(\rightarrow\) \(c \sim 10^{79}\)
Outline

• Overview ✓
• Asymptotic Symmetries ✓
• Entropy
  – Cardy formula
    \[ S = \frac{\pi^2}{3} cT \]
• Generalizations and applications
At extremality, first law of thermodynamics becomes

\[ 0 = T_H dS = dM - \Omega_H dJ \]

So define conjugate potential for extremal variations

\[ dS = \frac{dJ}{T_L} \]

For Kerr,

\[ S = 2\pi J \rightarrow T_L = \frac{1}{2\pi} \]

Quantum state of a field on extreme Kerr has density matrix

\[ \rho = e^{-\hat{J}/T_L} \]
Plug central charge and temperature

\[ c_L = 12J \]
\[ T_L = \frac{1}{2\pi} \]

into the Cardy formula

\[ S_{CFT} = \frac{\pi^2}{3} c_L T_L \]

\[ S_{CFT} = \frac{2\pi J}{\hbar} = \frac{\text{Area}}{4} = S_{macro} \]
If we assume

\[ c_R = c_L = 12J \]

then the Cardy formula gives the correct near extremal entropy,

\[ S_{CFT} = 2\pi J + 2\pi \sqrt{\frac{c_R}{6}} E_R + \cdots \]

Summary: We have only found the chiral left half of the CFT, but there is evidence for right-movers which account for the entropy away from extremality.
Outline

• Overview
• Asymptotic Symmetries
• Entropy
• Generalizations and applications

What can we compute?
Other black holes

- 4d Kerr
  Guica, TH, Song, Strominger

- Higher dimensions
  Lu, Mei, Pope

- Asymptotic AdS
  various papers

- Charge
  TH, Murata, Nishioka, Strominger

- String theory (D0-D6, D1-D5, NS5) and Supergravity
  Azeyanagi, Ogawa, Terashima
  Nakayama
  Chow, Cvetic, Lu, Pope
  Lu, Mei, Pope, Vazquez-Poritz
  Chen, Wang
Greybody Factors

★ Extreme Kerr has $T_H = 0$, but it decays via superradiance into modes

$$\Phi \sim e^{im\phi - i\omega t} S_\ell(\theta) R(r)$$

with

$$0 < \omega < m\Omega_H$$

For small $\omega$,

Decay rate = $\Gamma_\ell(\omega) \sim (\omega - m\Omega_H)^{2\ell+1}$

★ This is a two-point function in the CFT

$$\Gamma \sim \int e^{-i\omega_R x^+ - i\omega_L x^-} <O O>$$

similar to:
Maldacena, Strominger '97
Greybody Factors

• What about large frequency?

\[ \Gamma = \frac{\sinh^2 2\pi \delta}{\cosh^2 \pi (m-\delta) + \cosh^2 \pi (m+\delta) + 2 \cos 2\pi \sigma \cosh \pi (m+\delta) \cosh \pi (m-\delta)} \]

\[ \delta \equiv \text{function of } m, \ell, M \]

• Gravity: Teukolsky and Press 1974

• CFT: work in progress!
  
  with W. Song and A. Strominger
Conclusion

• **Summary: Gravity on extreme Kerr is a CFT.**
  – Nothing exotic is necessary (but exotic black holes work too)
  – Applies to astrophysical black holes, eg GRS 1915+105

• **Open questions**
  – Beyond extremality
  – What can we calculate with the CFT?
    • greybody factors?
    • astrophysics (accretion, X-ray emission, etc.)?

• **Wide open questions**
  – What is the CFT?
  – What/where are the microstates?
What about SL(2,R)?

**AdS\(_3\) (Brown-Henneaux)**

Exact: \( SL(2, R)_L \times SL(2, R)_R \)

Asymptotic: Virasoro × Virasoro

**Kerr**

Exact: \( U(1)_L \times SL(2, R)_R \)

Asymptotic: Virasoro × ???
What about SL(2,R)?

Near-extremal entropy

★ The zero mode of SL(2,R)$_R$ is

$\zeta_0 = \partial_t$

★ Writing this in terms of the original Kerr coordinates suggests

$Q_0 \sim M^2 - J \equiv E_R$

★ If we assume

$c_R = c_L = 12J$

then the Cardy formula gives the correct near extremal entropy,

$S_{CFT} = 2\pi J + 2\pi \sqrt{\frac{c_R}{6}} E_R + \cdots$

★ Summary: We have only found the chiral left half of the CFT in Kerr/CFT, but we suspect that there are also right-movers which account for the entropy away from extremality
Assumptions

• For Kerr/CFT ("quantum gravity on NHEK is a CFT"), only assumption is:
  
  • A consistent UV completion of quantum gravity on NHEK exists

• For entropy, using the Cardy formula assumes:
  
  • Modular invariance
  • Sufficient but not necessary condition:

\[ T \gg c \quad (\text{ie, } \frac{1}{2\pi} \gg 10^{79}) \]

Uh-oh.

Same thing happens in string theory, but is explained by highly twisted sectors. Does something similar happen here?

Maybe – the mass gap is very small \( \sim 1/M^3 \). This suggests an effective description with small \( c \), large \( T \). More on this later.


5D 3-charge black hole
\[ S = 2\pi \sqrt{n_1 n_2 n_3} \]

String theory U-duality changes \( c, T \) with \( S \propto cT \) fixed

5d Kerr (or 4d Kerr-Newman) has near horizon isometries
\[ SL(2, R) \times U(1)_\phi \times U(1)_\psi \]

Two consistent choices of boundary conditions:

- First choice: \( U(1)_\phi \rightarrow \text{Virasoro with central charge} \)
  \[ c_\phi \sim J_\phi \]

- Second choice: \( U(1)_\psi \rightarrow \text{Virasoro with central charge} \)
  \[ c_\psi \sim J_\psi \]

Either choice gives the correct entropy!