ASPECTS OF MIRROR SYMMETRY FOR ORBIFOLD CFTs ON K3

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W. NAHM/K.W.: „Mirror Symmetry on Kummer Type K3 Surfaces", hep-th/0106104

K.W.: „Orbifold Constructions of K3: A Link between Conformal Field Theory and Geometry", hep-th/0112006

2dim. $N=2$ SCA with central charge c:

energy momentum tensor T:

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2}T(w) + \frac{1}{z-w}\partial T(w)$$

$u(1)$ current J $(h_J = 1)$:

$$J(z)J(w) \sim \frac{c/3}{(z-w)^2}$$

supercharges $G^\pm$ $(h_{G^\pm} = \frac{3}{2}, Q_{G^\pm} = \pm 1)$

$$G^+(z)G^-(w) \sim \frac{2c/3}{(z-w)^3} + \frac{2}{(z-w)^2}J(w) + \frac{1}{z-w}\left(2T(w) + \partial J(w)\right) \sim -G^-(w)G^+(z)$$

MIRROR AUTOMORPHISM:

$$(T, J, G^\pm) \mapsto (T, -J, G^\mp)\,; \quad (\bar{T}, \bar{J}, \bar{G}^\pm) \mapsto (\bar{T}, \bar{J}, \bar{G}^\pm)$$

MIRROR SYMMETRY:

An equivalence of $N=(2,2)$ SCFTs which induces the mirror automorphism on the SCA.

Free field realization of N=2 SCA:

$c = \frac{3d}{2}$, $d \in 2\mathbb{N}$

$j^1, \dots, j^d$ generators of $u(1)^d$ (on d free bosons)

$\psi^1, \dots, \psi^d$ Majorana fermions

$$T = \frac{1}{2} \sum_{k=1}^{d} :j^k j^k: + \frac{1}{2} \sum_{k=1}^{d} :\psi^k \partial \psi^k:$$

$$\psi_\pm^{(k)} := \frac{1}{\sqrt{2}} (\psi^k \pm i \psi^{k+d}), \quad j_\pm^{(k)} := \frac{1}{\sqrt{2}} (j^k \pm i j^{k+d})$$

$$J = \sum_{k=1}^{d/2} :\psi_+^{(k)} \psi_-^{(k)}: ; \qquad G^\pm = \sqrt{2} \sum_{k=1}^{d/2} :\psi_\pm^{(k)} j_\mp^{(k)}:$$

The mirror automorphism can be induced by

$$\left. \begin{array}{l} \psi^k \mapsto -\psi^k \\ j^k \mapsto -j^k \end{array} \right\} \text{ for } k \leq d.$$

$\mathcal{N}=(4,4)$ SCFT with $c=6$

T: torus, $\dim_{\mathbb{C}} T = 2$; X : K3 surface

$$\mathcal{M}^{\text{tori}} = O^+(H^{\text{odd}}(T,\mathbb{Z})) \backslash O^+(H^{\text{odd}}(T,\mathbb{R})) / SO(4) \times O(4)$$

$\Downarrow$ MIRROR SYMMETRY can be induced by fiberwise T-duality

$\downarrow$ TRIALITY [Nahm/W.'99]

$$\Big(O^+(H^{\text{even}}(T,\mathbb{Z})) \backslash O^+(H^{\text{even}}(T,\mathbb{R})) / SO(4) \times O(4) \Big)^G$$

$\Downarrow$ "MIRROR SYMMETRY" $\Uparrow$

$\downarrow$ G-ORBIFOLD [Nahm/W.'99, W.'00]

$$\mathcal{M}^{K3} = O^+(H^{\text{even}}(X,\mathbb{Z})) \backslash O^+(H^{\text{even}}(X,\mathbb{R})) / SO(4) \times O(20)$$

[Seiberg'88, Cecotti'91; Aspinwall/Morrison'94]

ORBIFOLD LIMITS OF K3

$$\begin{array}{ccl} & T & \text{algebraic } G \text{ action, } G \subset SU(2) \text{ finite} \\ & \downarrow & \\ X \xrightarrow[\text{resolution}]{\text{minimal}} & T/G & \supset \mathcal{S} \text{ singularities of ADE type} \end{array}$$

e.g. $G = \mathbb{Z}_M$, $M \in \{2,3,4,6\}$:
$\qquad \xi$: $T =$ 

minimal resolution of A_{M-1} type singularity $s \in \mathcal{S}$:

$$\hat{E}_s^{(1)} - \hat{E}_s^{(2)} - \bullet - \cdots - \hat{E}_s^{(M-1)}$$

$\hat{E}_s^{(i)}$: Poincaré duals of irreducible components of exceptional divisors in the resolution

span root lattice $\hat{\mathbb{E}}_s$, $\quad \hat{\mathbb{E}} := \bigoplus_{s \in \mathcal{S}} \hat{\mathbb{E}}_s \subset H^2(X, \mathbb{Z})$

THE MIRROR MAP FOR TORI

$e_1,\dots,e_4$: standard $\mathbb{Z}^4 \subset \mathbb{R}^4$ generators, $\mathbb{R}^4 \cong H^1(T,\mathbb{R})$; $v \in H^4(T,\mathbb{Z})$, $v^0 \in H^0(T,\mathbb{Z})$ generators

$T_0 = \mathbb{R}^4 / \bigoplus_{i=1}^{4} r_i \mathbb{Z}$,

$\mu_i = \frac{1}{r_i} e_i$,

$$\gamma(T_0): \begin{cases} v^0 \mapsto \mu_1\wedge\mu_2 \mapsto -v^0 \\ v \mapsto \mu_3\wedge\mu_4 \mapsto -v \\ \mu_1\wedge\mu_3 \mapsto \mu_2\wedge\mu_3 \mapsto -\mu_1\wedge\mu_3 \\ \mu_4\wedge\mu_2 \mapsto \mu_1\wedge\mu_4 \mapsto -\mu_4\wedge\mu_2 \end{cases}$$

T_0': $\mathbb{Z}_3$ symmetric torus,

$\mu_1 = e_1$, $\mu_2 = \frac{1}{2} e_1 + \frac{\sqrt{3}}{2} e_2$,

$\mu_3 = e_3$, $\mu_4 = \frac{1}{2} e_3 + \frac{\sqrt{3}}{2} e_4$,

$$\gamma(T_0'): \begin{cases} v^0 \longmapsto \mu_1\wedge\mu_2 \longmapsto -v^0 \\ v \longmapsto \mu_3\wedge\mu_4 \longmapsto -v \\ \mu_1\wedge\mu_3,\ \mu_2\wedge\mu_3 \quad \text{invariant} \\ \mu_4\wedge\mu_2,\ \mu_1\wedge\mu_4 \end{cases}$$

Mirror map γ for $\mathbb{Z}_M$ orbifold limits of K3

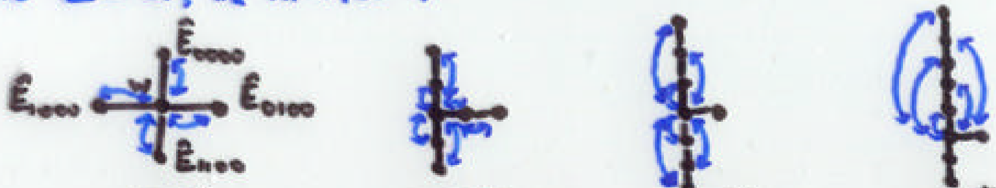
$\mathbb{E} = \bigoplus_{s\in\mathcal{F}} \mathbb{E}_s$, $\mathbb{E}_s = \mathrm{span}_{\mathbb{Z}}\{E_s^{(\ell)};\ \ell\in\{1,\dots,n_s-1\}\}$

$$E_s := \sum_{\ell=1}^{n_s-1} \ell\, E_s^{(\ell)}, \qquad E_s^{(0)} := -\sum_{\ell=1}^{n_s-1} E_s^{(\ell)}$$

$s = (s_1, s_2)$ with $\begin{cases} s_i \in \mathbb{F}_2 & \text{for } \mathbb{Z}_2, \mathbb{Z}_4 \text{ type fixed pts.} \\ s_i \in \mathbb{F}_3 & \text{for } \mathbb{Z}_3 \text{ type fixed pts.} \\ s_i = 0 \in \mathbb{F}_2 \cup \mathbb{F}_3/\sim & \text{for } \mathbb{Z}_6 \text{ type fixed pts.} \end{cases}$

$$\boxed{\gamma(E_s^{(L)}) = -\frac{1}{M} \sum_{\substack{t\in\mathbb{F}_2\cup\mathbb{F}_3/\sim \\ \gcd(n_t, n_{s_2})\neq 1}} \ \sum_{\substack{0\leq \ell < M, \\ \ell \equiv L \bmod n_s}} \theta^{\ell\cdot t\cdot s_1}\left(E_{(t,s_2)}\right) \qquad \text{ord}(\gamma):\ 4, 12, 8, 12 \text{ if } M:\ 2, 3, 4, 6}$$

up to $\mathbb{E}\otimes\mathbb{G}$, $\pi_*(H^2(T,\mathbb{Z})^G)$:



I_0^* $\quad$ IV^* $\quad$ III^* $\quad$ II^*

The mirror map $\mathcal{F}$ for $\mathbb{Z}_M$ orbifold CFTs on K3

$s=(s_1,s_2)\in\mathcal{S}$, $s_i\in\mathbb{F}_2\cup\mathbb{F}_2/n_s$, $\ell\in\{1,\dots,n_s-1\}$:

$$\mathcal{F}(T_s^{\ell}) = \sum_{\substack{t\in\mathbb{F}_2\cup\mathbb{F}_2/n\\ n_{s_1}\mid \ell n_t}} a^{\ell}_{n_{s_1},n_{s_2}}\, T_{(t,s_1)}^{\ell n_t/n_{s_1}}\, \zeta_{2n_t}^{\ell t\cdot s_1},$$

$$a^{\ell}_{n,n'} = \sqrt{\tfrac{n}{n'}}\left(1-\zeta^{\ell}_{n'}\right)^{-1},$$

$$\zeta_{n'} = e^{2\pi i/n'}$$

RESULTS [Nahm/W'01]

- explicit calculation of mirror map $\gamma \in O^+(H^{even}(X,\mathbb{Z}))$ as induced by fiberwise T-duality for $\mathbb{Z}_M$ orbifold limits X of K3
- for $\mathbb{Z}_M$ orbifold CFTs on K3: mirror symmetry acts by fiberwise $\mathbb{Z}_M$ type discrete Fourier transform $\mathcal{F}$ on the twisted ground states
- map C with $C\gamma = \mathcal{F}C$ gives a CFT realization of the McKay correspondence which is compatible with the Weyl algebra representation of the vertex operator algebra on the underlying toroidal theory.
- for $(\widetilde{\mathbb{Z}})^4 = (\mathbb{Z})^4/\mathbb{Z}_2^3$: our mirror map agrees with the one given by Greene and Plesser [Greene/Plesser '90] and the semitoric approach [Batyrev '94, Aspinwall/Morrison '94, Dolgachev '96, Rohsiepe '01] (WN)